Media and Machines Lab
Washington University in St. Louis
School of Engineering & Applied Science

Surface Network Construction from Non-parallel Cross-sections

Lu Liu, Tao Ju

Introduction

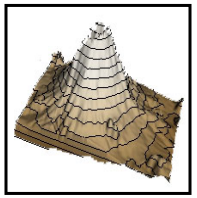
Problem

Given a set of curve networks on 2D cross sections, construct a surface network interpolating the curves that is not only **topologically correct** but also **geometrically smooth**.

Motivation

Surface-from-curve methods have numerous applications, including:

- Building terrain geometry from height curves.
- Delineating anatomical structures from medical images.

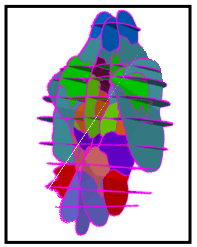


Terrain from height curves

Challenges

Due to advance in imaging technique and software tools, today's curve data can be very complex (see the example on the right):

- Cross-section planes can be **arbitrarily orientated**.
- Each cross-section can have a **curve network** instead of a simple closed curve.



Non-parallel cross-sections of a mouse brain. Each cross-section contains a network of boundary curves of 10 anatomical regions.

Previous work

Abundant research has been done in last three decades. However, most of them can only handle parallel cross-sections. Representative methods are:

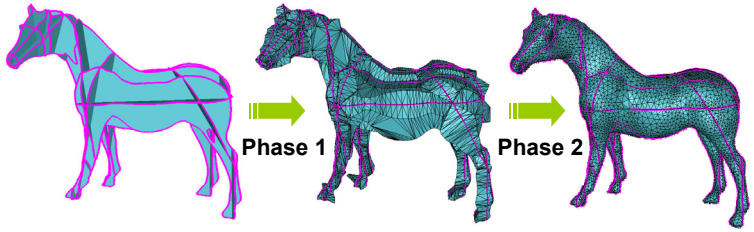
- Polygon tiling
- Delaunay meshing
- Implicit interpolation
- Projection based methods

While few methods are designed to handle non-parallel cross-sections [Boissonnat 07], they are limited to only simple closed curves on each plane.

★ To the best of our knowledge, so far no method can handle complex curve networks on non-parallel cross-sections.

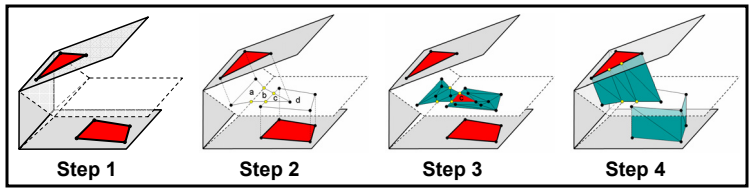
Method

Method overview



Phase 1: Initial surface construction

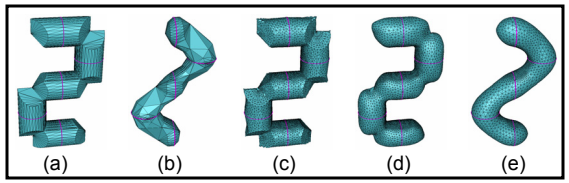
The first phase constructs a topologically correct surface from the input curve-networks. The idea is simple: consider a partitioning of the space by the input planes, create surface within each sub-space and stitch them together. To build a surface within each sub-space, we follow the four steps as illustrated in the picture below:



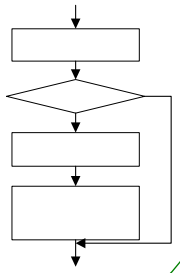
Step 1: Generate the medial axis of the sub-space
Step 2: Project curve networks onto MA sheets
Step 3: Triangulate regions on MA sheets projected from different materials
Step 4: Connect curve networks with their projections on the MA

Phase 2: Surface fairing and refinement

While the first phase builds a topologically correct surface, the geometry is unsatisfactory. In particular, it is not smooth and contains triangle with widely varying shapes and sizes. The second phase resolves the problem by iterating mesh fairing with mesh refinement.

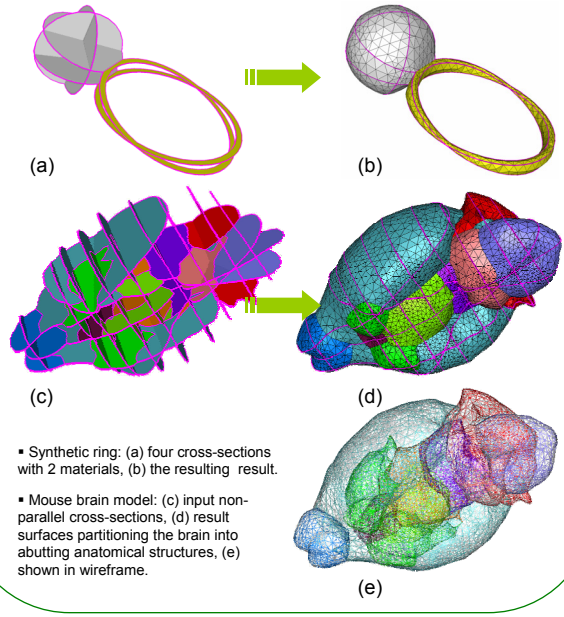


Fairing and refinement: (a) initial surface (b) only apply refinement (c) only apply fairing (d) fairing after refinement (e) result of our iterative algorithm



Results

Examples



- Synthetic ring: (a) four cross-sections with 2 materials, (b) the resulting result.
- Mouse brain model: (c) input non-parallel cross-sections, (d) result surfaces partitioning the brain into abutting anatomical structures, (e) shown in wireframe.

Conclusion

Features of our method

★ Handles a wide range of inputs
  - planes of cross-sections with arbitrary orientations
  - curve networks with arbitrary shape and topology
★ Guaranteed topological correctness
★ Geometrical smoothness

Current and future work

- So far there is no publicly available package for performing surface construction from arbitrarily oriented planar curves. We are working on writing such first package in C++.
- We assume that the input data are closed curve networks on the input cross-sections which are consistent with each other. Our future work is handling inconsistent and incomplete data and allowing user-interaction to change the topology of the surface.