

FAIR DIVISION IN THEORY AND PRACTICE

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Course description

The concept of fair division is a central tenet in the design of procedures aimed at generating equitable social outcomes and mitigating conflict. At the national level, such procedures include systems of apportionment, voting, and legislative districting, to name a few. On a smaller scale, these procedures could govern how assets are divided in a divorce, or how to divide a cake.

While “fairness” in theory is indisputably a good thing, in practice the courts, politicians, and even mathematicians have grappled with the question of what it means for a procedure to be fair. An illustration is the 2004 case *Vieth v. Jubelirer*, in which a plurality of the Supreme Court ruled that claims of political gerrymanders (the drawing of district lines to advantage one political party over another) were not capable of being decided by a court. Interestingly, the Court unanimously agreed that severe gerrymanders are unconstitutional and incompatible with democratic principles of fairness. But neither the Court nor the appellants could agree on a principle of fairness that could be applied to all such cases. Absent such a principle, Justice Scalia wrote “ ‘Fairness’ does not seem to us a judicially manageable standard.”

This course will examine algorithms and applications of procedures that aim to divide or allocate resources fairly. Some of these procedures were developed by mathematicians looking for formulas that satisfy mathematical properties such as *envy-freeness* and *equitability*. Other procedures emerged through historical debates concerning issues like federalism and voting rights. Still others emerged as solutions to ethical and practical problems such as how to fairly allocate kidneys, or match students to schools. All of the procedures we consider will be examined in terms of the fairness goals they aspire to achieve, the mechanisms they employ to achieve those goals, and the shortcomings of the procedures.

Structure of the course

This course spans the fields of political science and computer science. Each “topic” will be presented as a module with a lecture concerning the applicability of the procedure to questions in political science and a hands-on lab in which students will implement the procedure themselves. The goal is for students to emerge with an understanding of the practical problem the procedure was meant to resolve along with the mechanics of the procedure and its strengths and shortcomings.

Grading: You'll be graded on regular problem sets (5%), a first exam (30%), a second exam (30%), and a project that you will present to the class during our "science fair" (30%). The remaining 5% of your grade will be based on your participation in class and attendance.

Readings: Readings are listed on the syllabus by topic. If the material is not available on JSTOR we will provide a link to it on the course website. We also recommend the books *Fair Division: From Cake-Cutting to Dispute Resolution* by Brams & Taylor, and *The Mathematics of Voting and Elections* by Jonathan Hodge and Richard Klima.

INTRODUCTION

- Description of course, syllabus, schedule, programming and mathematical notation.

TOPIC 1

- "Divide-and-choose," and its variants.
- Excerpts from *Fair Division: From Cake-Cutting to Dispute Resolution*, Brams & Taylor, 1996.

TOPIC 2

- The Adjusted Winner procedure
- "Fair Division, Adjusted Winner Procedure (AW), and the Israeli-Palestinian Conflict," Massoud, *Journal of Conflict Resolution*, 44(3):333-358, 2000.
- Excerpts on the Panama Canal treaty negotiations, in *Fair Division: From Cake-Cutting to Dispute Resolution*, Brams & Taylor, 1996.

TOPIC 3

- Apportionment in the U.S. House: Fair allocation of seats to states, and axioms of proportionality
- Excerpts from *Fair Representation: Meeting the Ideal of One Man, One Vote*, Balinski and Young, 1982.
- *House of Representatives Apportionment Formula: An Analysis of Proposals for Change and Their Impact on States*, Royce Crocker, Congressional Research Service Report for Congress, 2010.

TOPIC 4

- Measures of proportionality with an application to the list systems of proportional representation
- “Proportionality, Disproportionality and Electoral Systems,” Michael Gallagher, *Electoral Studies*, 10(1): 33-51, 1991.

TOPIC 5

- Approval voting, the cumulative vote, and the bloc vote: Disproportional allocation of multiple seats
- “Cumulative Voting in the United States,” Pildes and Donoghue, *University of Chicago Legal Forum* 241, 1995.
- “Race and Voting Rights in Ferguson,” *New York Times*, January 4, 2015.

TOPIC 6

- Positional and transferable vote systems
- Chapter 10 in *Fair Division: From Cake-Cutting to Dispute Resolution*, Brams & Taylor, 1996.
- “The Winning Strategy in Oakland: Concentrate on Being 2nd or 3rd Choice” *New York Times*, November 11, 2014.

TOPIC 7

- Shapley-Shubik and Banzhaf power indices: Weighted votes for differently-sized constituencies
- “Weighted Voting in New York,” Grofman and Scarrow, *Legislative Studies Quarterly* 6(2): 287-304, 1981.

TOPIC 8

- Arrow’s Theorem and Sen’s Theorem: Sensibility in procedure and outcome
- Excerpts from *Social Choice and Legitimacy: The Possibilities of Impossibility*, Patty and Penn, 2014.

TOPIC 9

- The Gibbard-Satterthwaite Theorem: Non-manipulability of procedures
- “Arrow’s theorem and the Gibbard-Satterthwaite theorem: a unified approach,” Philip Reny, *Economics Letters*, 70(1): 99-105, 2001.
- “Almost all social choice rules are highly manipulable, but a few aren’t,” Jerry Kelly, *Social Choice and Welfare*, 10(2):161-175, 1993.

TOPIC 10

- Matching Markets: School choice, organ donations, and marriage
- “College Admissions and the Stability of Marriage,” Gale & Shapley, *American Mathematical Monthly*, 69(1):9-15, 1962.

TOPIC 11

- Gerrymandering: Measuring district shape and composition
- “The Quest for Legislative Districting in the Public Interest: Elusive or Illusory,” Lowenstein and Steinberg, *UCLA Law Review*, 33(1), 1985.
- “Gerrymandering and convexity,” Hodge, Marshall & Patterson, *The College Mathematics Journal*, 41(4): 312-324, 2010.

TOPIC 12

- Measures of segregation: Evenness, dispersion and contact as fairness criteria
- “Sorting Out Deepening Confusion on Segregation Trends,” Orfield, Siegel-Hawley & Kucsera, *The Civil Rights Project*, March 2014.

IMPORTANT DATES

FIRST EXAM:	February 27
SECOND EXAM:	April 15
IN-CLASS PRESENTATIONS:	April 20, 22
POSTER SYMPOSIUM:	April 24