

# Fair Division in Theory and Practice

Ron Cytron (Computer Science)

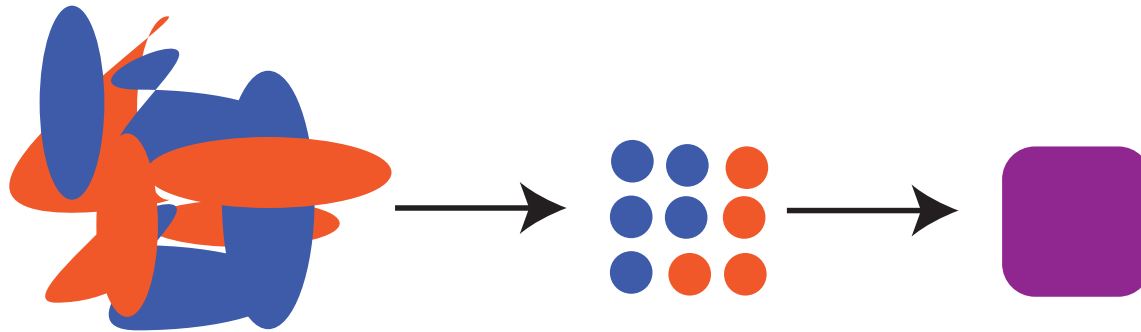
Maggie Penn (Political Science)

*Introduction*

## The fair division problem

- How to allocate objects (or resources) between people (or groups) who are entitled to them
- In politics:
  - Dividing up tangible resources  
Contested territories; budgets; voting populations; “cake”
  - In representative government, dividing up “influence”  
Which interests are represented?

## The classic problem of democratic representation



Given a distribution of interests, how do we map that distribution into a set of representative seats?

OR how do we make political choices that are representative of the interests they're meant to further?

## A seemingly benign example: Apportionment

- There are 435 seats in the U.S. House of Representatives
- Constitutionally they are to be divided between the states on the basis of the states' populations
- “State interests” to be represented proportionally
- How should this division occur so as to be *most fair*?

## A possibly less benign example: Proportional representation (PR)

- In previous example, we distributed a set of *seats* to *states* on basis of population
- In PR systems, we distribute *seats* to *parties* on basis of their support amongst the electorate
- “Population” as an entitlement measure is straightforward
- “Support” amongst an electorate is less so
  - What is the goal of these systems? To produce a government that perfectly mirrors the electorate’s preferences? To simply ensure that every group has some representation in government? How do we know that the parties running represent these true preferences?

## Issues of fair division in non-PR systems

- In many school board elections, multiple members are chosen concurrently
- Ferguson/Florissant: 7 school board seats intended to represent interests of community
- How can we determine if a procedure to allocate these seats is fair? (The ACLU is currently suing the district)

“At the end of the day, if the parents of 77 percent or more of the students who are being educated in the Ferguson-Florissant School District cannot have a larger representation on this school board, something is wrong with the voting process in Ferguson.”

(St. Louis NAACP President Adolphus Pruitt)

## Weighted voting in New York

- NY state had many voting bodies composed of people who represented differently-sized constituencies
  - Eg. The mayors of two cities might serve on a county board
- Fair division problem: How to allocate voting weights across the board members to ensure that their “power” was proportional to the size of the constituencies they were representing
- What is “voting power”? If I have 25% of the total voting weight, do I have 25% of the power?

## Court case showing weighted voting was unfair

### Nassau County Board's Voting System

City	# Votes
Hempstead 1	9
Hempstead 2	9
North Hempstead	7
Oyster Bay	3
Glen Cove	1
Long Beach	1

30 Total votes, 16 required to pass

Do you see a problem?



## How to choose a procedure?

There are an infinite number of procedures that we could use to allocate resources

What might we want these procedures to satisfy?

- Give each person equal say in the selection process
- Not procedurally advantage one outcome over another
- Florissant / weighted voting example: Neutral formulas don't necessarily produce fair outcomes!
- *Ex ante* versus *ex post* fairness
- The fact that procedural choice is meaningful—different procedures choose different things—implies that we need to think about how procedures affect distributive outcomes
- This class: we will think about quantifying procedures and outcomes in terms of the fairness goals we aspire to