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Cake Cutting Project

My project is about assigning household chores to 3 or more people using the Last Diminisher algorithm.

Each person rates each chore with a single number on a scale of 1 to 10 where 10 is the most undesirable. Ratings are made specifically on desirability--as opposed to difficulty--to encapsulate frequency and difficulty together. When a chore is assigned to a person it is assumed that they are responsible for completing it however often it recurs.

With this scheme you cannot request pairs of chores (wanting to clean all bathrooms or none). Also, there are several common chores that some household members cannot be assigned due to difficulty (Marie is not old enough to use the lawnmower) or time of day they must be done (Melody can't do lunch dishes because she has class at that time).

The players are ordered at random, and the first person, p0 makes a chore list that they view to be proportional. This done by trying to create a list of chores whose sum is 1/n rounded up to the nearest integer, and if that fails, incrementing the tolerance and attempting to create a list of sum (1/n)-tolerance, or (1/n)+tolerance. This violates proportionality up to the value of one chore, but that is to be expected with indivisible items.

Attempting to make a subset of a collection whose sum is of a certain value is the NP Complete Knapsack Problem, and so (presumably) the fastest this can be done is in pseudo-polynomial time with a dynamic programming approach. With the very small numbers I'm working with, this is a non-issue.

After p0 has made a proportional list of chores, the next player p1 looks at the list and determines if it is difficult enough to be proportional. If p1 thinks it is difficult enough, p1 passes the list to p2. If p1 thinks the list is too easy, p1 will add add chores from the original list (that are not in the list in question) using the same technique as when making the first list.

When two players remain, the list is divided using Divide and Choose.

A person who receives their list later may be envied by someone who received their list earlier, because only as long as a player remains in the game can they add to lists to ensure that no one else gets a list that is too easy.

Ratings

I got a list of chores from my mother Marina which was to be divided between her and three of my sisters Melody, Megan, and Marie. Here are the ratings they recorded.

	Megan	Marie	Melody	Marina
Clean downstairs bathroom	6	1	2	10
Clean main level bathroom	4	1	3	5
Clean upstairs bathroom	7	1	7	1
Clean parents' bathroom	6	1	7	1
vacuum upstairs hallway	8	4	7	5
vacuum upstairs little girls room	8	5	7	7
vacuum upstairs master bedroom	8	8	7	5
clean master bedroom mirrors	3	2	1	10
Tidy deck	8	4	6	8
Dejunk + Vaccuum Minivan	9	8	10	5
Vacuum Family room rugs and runners	6	3	5	10
Sweep Kitchen	5	4	8	10
Empty Dishwasher	6	4	5	10
Load Dishes after dinner	8	4	9	10
Scrub Pots and pans after dinner	9	1	10	10
wipe counters and stove after dinner	8	1	4	10
clean kitchen cabinets	6	5	10	8
Set table	3	3	4	10
Walk dog	4	1	1	1
Fold Laundry Load	5	1	8	5
Wash Laundry Load	4	4	2	1
Vacuum stairs	5	2	5	10
Dust dining room	4	5	3	10
Dust family room	5	5	2	10
Wipe Kitchen appliances	4	1	1	10
Tidy Family Room	5	8	7	10

Clean kitchen computer desk	6	5	2	5
wipe out refrigerator shelves	9	9	10	3
sweep garage	6	7	2	10
scoop poop in yard	5	10	10	10
mop kitchen	9	9	10	10
mop wood floors	8	9	10	10
sweep wood floors	4	5	2	10
take out compost	9	2	7	10
take out trash to bin	5	2	2	10
take trash to curb and back	8	2	4	5
take recycling to curb and back	8	2	4	5
mow lawn	10	10	4	10
trim lawn	10	10	10	10
wipe kitchen windows	7	3	2	10
restock paper goods	3	2	2	10
vacuuming downstairs couches	6	8	9	10
wipe kitchen table	3	1	4	10
wipe kitchen chairs	8	7	9	10
clean microwave	8	5	1	5
wash car	9	10	9	5
clean light fixtures	8	3	1	10
Load lunch dishes	8	3	7	10
Empty dishwasher before dinner	9	3	8	5
Load breakfast dishes	9	3	4	5
Clean table after breakfast	4	1	2	5

Results

After running the algorithm, these lists were assigned:

Megan
mop wood floors
take out compost
take trash to curb and back
take recycling to curb and back
mow lawn
trim lawn
restock paper goods
vacuuming downstairs couches

Marie
Clean downstairs bathroom
Clean main level bathroom
Clean upstairs bathroom
Clean parents' bathroom
vacuum upstairs hallway
vacuum upstairs little girls room
vacuum upstairs master bedroom
clean master bedroom mirrors
Tidy deck
Dejunk + Vaccuum Minivan
Vacuum Family room rugs and runners
Sweep Kitchen
Empty Dishwasher
Load Dishes after dinner
Scrub Pots and pans after dinner
wipe counters and stove after dinner

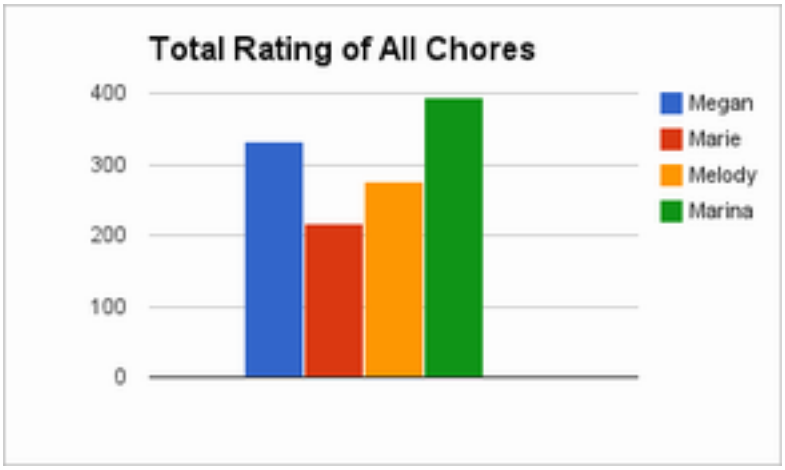
Melody
clean kitchen cabinets
Walk dog
Fold Laundry Load
Vacuum stairs
Dust dining room
Dust family room
Sipe Kitchen appliances
Tidy Family Room
Clean kitchen computer desk
sweep garage

scoop poop in yard
mop kitchen
Wash Laundry Load
sweep wood floors
take out trash to bin
wipe kitchen windows

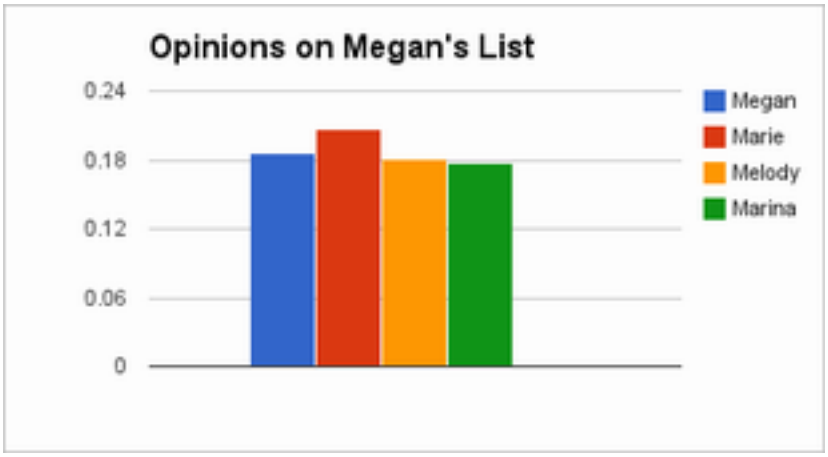
Marina
wipe kitchen table
wipe kitchen chairs
clean microwave
wash car
clean light fixtures
Load lunch dishes
Empty dishwasher before dinner
Load breakfast dishes
Clean table after breakfast
wipe out refrigerator shelves

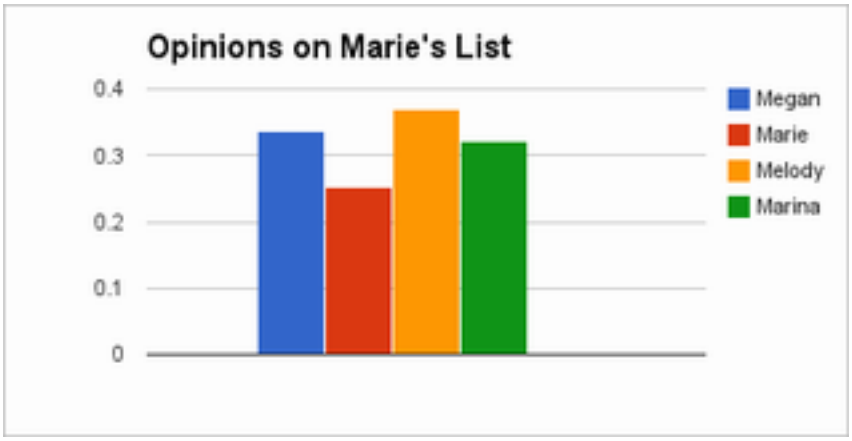
Findings From Real Preferences

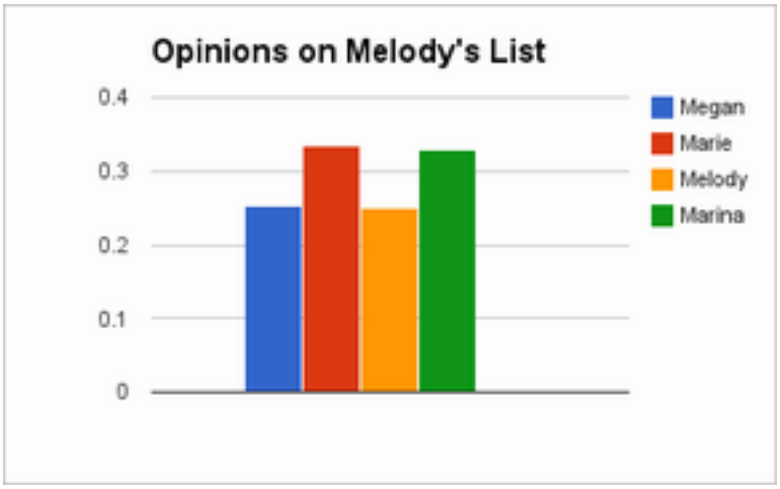
This graph shows the sum of all ratings for each person.
Interestingly, the ambitious 5 year old had the lowest sum and the mom had the highest.

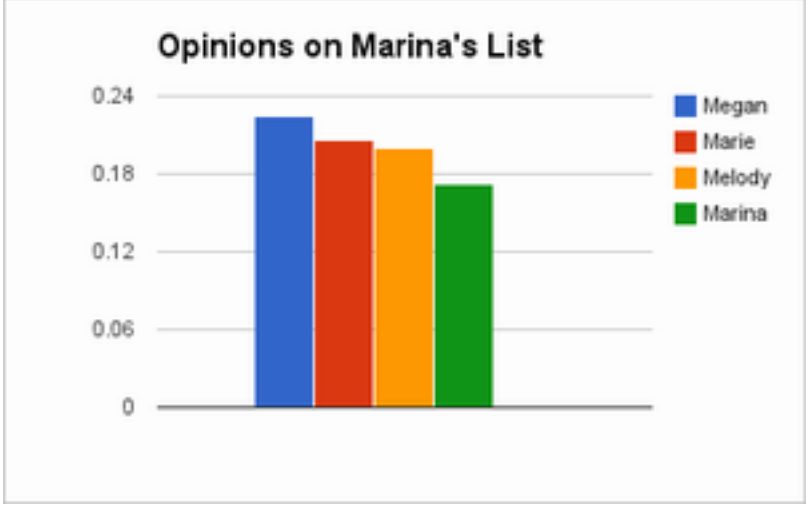


The next four graphs show how each person values each list as a fraction of their total rating of all chores.




Opinions on Marie's List
0.4
0.3
0.2
0.1
0
Megan
Marie
Melody
Marina


Opinions on Melody's List
0.4
0.3
0.2
0.1
0
Megan
Marie
Melody
Marina


Opinions on Marina's List
0.24
0.18
0.12
0.06
0
Megan
Marie
Melody
Marina

By the numbers:

Megan: Envies no one.
Marie: Envies Megan and Marina.
Melody: Envies Megan and Marina.
Marina: Envies no one.

But when asked about the lists that they were assigned:

Megan: Thought her list seems a little harder than proportional, and that Marina got an easy list.
Marie: Thought her own list was too hard, and Megan and Marina got lists that were too easy.
Melody: Thought Marina's list was a bit easy, but did not want to swap for it.
Marina: Did not envy anyone, thought Marie received a list that was too hard.

Informally, running the algorithm multiple times resulted in assignment of 80-90 percent of the total work perceived by all the players. Largest deviation from proportionality was less than .01 while greatest envy was less than 0.1.

Conclusions

Although there were some chores that could not be assigned to some people, this was not an issue. The players rated the chores they were unable to do as very undesirable which made it very unlikely they would receive them.

In general, the players had a hard time with assigning just one number for the subjective desirability of a chore, and could not throw out the concept of objective difficulty. Melody's comments illustrated this when she said that Marina's list looked easier, but she didn't want it.

The biggest issue was that 3 out of the 4 people were inconsistent in their ratings and evaluations of the final lists. The assumption that a list is the sum of its parts did not hold, and unfortunately the ratings of individual chores did not predict the overall reaction to an assignment.

Overall, Last Diminisher is not a great fit for this type of division. While qualitative assessments of chores and lists come quite easily, quantitative assessments do not. This algorithm provides a decent starting point from which players can trade chores. However, if I were to start over, I would choose a Balanced Alternation algorithm to avoid forcing players to quantify preferences.