Special Topics in Computer Science Theory
Team members: Chien Le/ Xiaoxuan Sun/ Chong Li
Repo name: Property Division

A brief one paragraph description of your project:

- Our project mainly focuses on dividing the indivisible properties. This method is applied in many situation of the real life such as: divorce, inheritance, etc. We try to find the best method which will help to divide properties to certain number of players so that it is fair share and envy-free situation. After the group discussion, we decide to set up the scenario where roommates in the apartment share and use internet for specify time slot as they want and win through bet. If person A wants to have specified time slot, A needs to bet and wins the time slot. In order for person A to get the specified time slot which he desires, he needs to trade it with something (in this project, he needs to pay by cashes for others). If there is a time slot where people want to use at the same time, bandwidth will be share equally to people in the apartment. The internet bandwidth is divided based on the probabilities from the bet between people.

An extended description of the project:

- In our project, we decide that every day is divided into 24 time slots. Each person in the apartment need to fill out the time slots and choose which time slots they want. Then, all notes of time slots are collected and compared by outside person. If there are time slots which no one claims, internet bandwidth will be divided by number of players so each player will have fair share of internet bandwidth.
- There are two approaches which we try to do in order to divide time slots for players fairly:
    - First approach: if the time slots are chosen by only one person at the time, it will belong to that person (for example, person A wants to have time slot from 6AM-8AM and no one else wants it, then that time slot is belonged to A) and he needs to trade in something (cashes, foods, etc.) in order to use entirely internet bandwidth during those time slots. If there are 2 or more people who want specified time slot, they need to bet and decide who will get that time slot.(There are few factors in which affects to the reason of use for time slots such as: important of tasks which players want to use that time slot, etc.)If there are two or more people who want to use one time slot at the same time, then people will bid to find the probability between each person. Then, we will share bandwidth of internet which is based on probability. For example, we have 2 people (A and B) who want to use time slot from 6PM to 9PM at the same time, and they are in need to use it. Then, we will let A and B bid on that time slot (given that A bid 6 and B bid 8 on the same time slot). We will have A's probability is 6/14 and B's probability is 8/14. In the end, we apply those probabilities and give right rate of internet bandwidth for each person. We assume that internet bandwidth is

100, then A will have 42.86 rate of internet bandwidth and B will have 57.14 rate of internet bandwidth.

- Second approach: in this approach, Advanced Knaster Sealed Bid happens 24 times at most. At the beginning, players should have the same amount of credits in order to bid for the time slot. Players should know when and how many time slots which they want to use throughout the day and the percentage of balance he uses to bid for each time slot. If there is only one player bid at certain time slot, that time slot will belong to that player and the only player won't need to pay. On the other hand, if there are two or more players at the same time slot, we will check first, if the sum of three players' bandwidth is equal or less than total internet bandwidth, they will share this time slot and need pay nothing, otherwise, we will have a special bid to allocate this time slot to highest bid player (s), like Player A bids 100 credits, Player B bids 100 credits, Player C bids 90 and their bandwidth needs are 15,15,15,(total internet bandwidth is 30), in this case, Player A and Player B can share this time slot and pay credit to Player C who didn't win this time slot.

- Applied algorithms:
  - For the first approach, we want to apply moving knife algorithm (Dubins and Spanier algorithm). On the other hand, we also want to apply Knaster sealed bid algorithm in order to find who will get time slots and how much he/she will pay for that time slots. The amount of money in which he needs to pay for time slots is based on the money from the bet. [However, this algorithm is fair-share and envy-free for two players but it will no longer envy-free if there are more than two players. Therefore, we also try to find other methods in order to solve this problem.]. For special case, if there are 2 or more people want to use time slot at the same time, we will apply normal probabilities to divide internet bandwidth for each person.
  - For the second approach, we will use the optimized Knaster Sealed Bid algorithms. Basically, we will apply Knaster Sealed Bid algorithm with two main changes in the algorithm. First change is that each player will bid credit at each time slot instead of giving complete references for all time slots at the beginning. Second change is that time slot can be shared between players instead of belonging to only one person.

A description of the approach you have taken in terms of modeling and/or code:

For our first approach:

-Knaster Sealed bid procedure for time slots (in this situation, there is no more than 1 person who really in need of the same time slot) see table below:

- Each player makes his/her (sealed) bids for each time slot.
- For each player, find the total dollar value of the bids on all the time slots.
- Compute each player's fair share by dividing each total just computed by the number of players.
- Give each time slot to the player who made the highest bid for that time slot.
- Compute the total value of the time slots won by each player (using the amounts of their winning bids).
- For each player, compute the difference between the player's fair share and the total value of the time slots he/she won.
- Compute the total amount of money received by the estate from the players (by adding/subtracting the first three entries on the 10th line of the accompanying table).
- The total paid to the estate is now divided equally and returned to each of the players.
- Summarize the final distribution of time slots and cash.

- Procedure for situation where there is more than 2 people in need of using the same time slot (for this procedure, we could not find any algorithms which fit with what we intend to do so we create simple algorithm [proportional divider] in order to solve case):

- Each of players decides which time slot is important to them.
- For each player, they are allowed to bid for the time slot which they want the most from range 0 to 100. (Rate also depends on player's "feeling" of how important that time slot will be for them).
- Add up rates from all players together to get SUM RATE.
- Take rate from each player and divide by SUM RATE which we found above, that will be rate of internet bandwidth for each player (we called it PERSONAL PROPORTIONAL.)
- Take Personal Proportional and multiple it with internet bandwidth to find out internet bandwidth rate for each players.
- Take different between proportional and give money to each player.

<u>For our second approach</u>:

- Optimized Knaster Sealed Bid:

- Each player should have the same amount of credit.
- All players need to submit time slots which they need to use to the referee.
- All players also need to give internet bandwidth at each time slot which they need to the referee.

- At the beginning of each time slot, a decision is made, naturally or by bid.
- There are 3 different cases:
    - Case 1 (no conflict): if there is no player or only 1 player wants to use internet at certain time slot. Therefore, that time slot will belong to that player.
    - Case 2 (conflict happens but all players can share internet): if the sum of references internet bandwidth from all players involved in conflict is less than or equal to actual internet bandwidth, then all players can share internet bandwidth at that time slot. There is no one needing to pay credit to get the time slot.
    - Case 3 (conflict happens and there are only few players can use the internet): if the sum of references internet bandwidth from all players involved in conflict is more than actual internet bandwidth, we do Knaster Sealed Bid in order to find players who can use internet.
      In sealed bid, a tricky situation is, 3 players attend bid, these 2 bid more money than the 3rd one can share the internet. We should take this into consideration, because internet is actually divisible item.
- After bid, the winner(s) should give the credits they used to bid to the loser(s). If there are multiple losers, credits are distributed according to the ratio of the credits of each loser. By doing so, the losers at this round could have more chance to win the next round. We call it approximately fair.

A description of the *analysis* methods you have used (e.g., simulation):

-For analysis methods, we collected data from one of our friends' apartment. There are two people who live in the same apartment and they have issue of using internet at the same time. The reason is both of them use application which requires high bandwidth internet (face chatting with families, online games, download services, etc.) We think that this situation is suitable with our case study so we collected data and ran experiments. We ran test on this only for one day. We divided time slots into 24 hours as described and let each players to have their decision on which time slots is important to them (no one knows other purpose of using time slots). Tables below will show preferences of two roommates (we assume that each player does not know other preference of time slots):

First person's preferences (we called first player is A):

12AM	1AM	2AM	3AM	4AM
5AM	**6AM**	**7AM**	**8AM**	**9AM**
			Online Homework **** (10)	Online Homework ****(10)
10AM	**11AM**	**12PM**	**1PM**	**2PM**
Online Homework ****(10)				
3PM	**4PM**	**5PM**	**6PM**	**7PM**
	Face chat Families ****(20)	Face chat Families ****(20)	Face chat Families ****(20)	Online Game Internet Surfing **(25)
8PM	**9PM**	**10PM**	**11PM**	
Online Game Internet Surfing **(25)	Online Game Internet Surfing **(25)	Online Game Internet Surfing **(25)	Online Game Internet Surfing **(25)	

Secondperson's preferences (we called second player is B):

12AM	1AM	2AM	3AM	4AM
5AM	**6AM**	**7AM**	**8AM**	**9AM**
				Online Shopping **(10)
10AM	**11AM**	**12PM**	**1PM**	**2PM**
Online Shopping **(10)	Audio Chat ***(20)	Audio Chat ***(20)		Homework *****(10)
3PM	**4PM**	**5PM**	**6PM**	**7PM**
Homework *****(10)	Homework *****(10)		Online Game *(25)	Online Game *(25)
8PM	**9PM**	**10PM**	**11PM**	
Online Game *(25)	Video Chat with Families ***(20)	Video Chat with Families ***(20)	Video Chat with Families ***(20)	

Notes: the * indicates how important task will be to player (it is scaled from 0 to 5 of *) and the number next to the * indicates how much internet bandwidth player will need for that time slot.

First approach: firstly, we apply moving knife algorithm to time slots which start at 12AM. As we can see from the preferences tables of two players, there are several slots which are not picked up by any players. For those empty-task time slots, internet bandwidth will be divided

equally 50/50 between two players. We continue applying moving knife algorithm in schedule. For time slot at 8AM, it will belong to A because B does not need it. For time slots at 9AM and 10AM, we need to apply Knaster sealed bid algorithm in order to find who will get these time slots. Comparing between A's need and B' need, we can see that A will bid higher in order to get those time slots for online homework. For time slots from 11AM-12PM, it will belong to B because A does not do anything on the internet. For the time slots 2PM-3PM, it will also belong to B because A does not use internet. The main problem occurs at time slot 4PM because player A and player B are in need of using that time slot for important tasks. In this case, we cannot use Knaster Sealed Bid algorithm to solve the problem because there will be only one person can have time slot to do their task.In this case, we need to use proportional divider algorithm in order to have both players to use internet at the same time. Each player will be allowed to rate their task which bases on the range of scale from 0 to 5. In this case, A gives his task with rate at 4 and B gives his task with rate at 5. Then, we will take both rate from both players and add them together, so we have SUM RATE = 5 + 4 = 9. After that, we take rate from each player and divide by SUM RATE to get each player's proportional [A will have proportional at 4/9 and B will have proportional at 5/9]. Next, we will take internet bandwidth and multiply by each player's proportional to have their own rate of internet bandwidth [At their apartment, they have internet bandwidth of 30Mbps. In the end, A will have (4/9)*30 = 13.33Mbps and B will have (5/9)*30 = 16.67Mbps.]. For time slot at 5PM, it will belong to A because B does not use internet at that time. For the time slot at 6PM, we apply Knaster Sealed Bid algorithms again in order to find who will get time slot and others will get money in exchange of not using internet. In this case, A will get time slot at 6PM and B gets paid by A. For time slots from 7PM to 8PM, both of players do not have important tasks to do. There are two choices for two roommates in order to use internet. First choice is both players can agree together that internet can be shared equally 50/50 for both players. Second choice is we can use proportional divider algorithms in order to find each player's internet bandwidth. For time slots from 9PM to 11PM, B is in need of using internet bandwidth for video chat with families. Therefore, we use Knaster Sealed Bid algorithms to find out how much B need to pay for A.

Second approach: each player should have 1000 credits at the beginning. A will have 11 time slots which he needs to use internet throughout the day. B will have 13 time slots which he needs to use internet throughout the day. For 8AM time slot, it will belong to A because B does not need to use internet at that time. For time slots from 9AM to 10AM, the sum of needed internet bandwidth is 10 (A) + 10 (B) = 20 which is less than the actual internet bandwidth rate (30), so A and B can share those time slots. For time slots from 11AM to 12PM and 2PM to 3PM, it will belong to B. The main problem occurs at time slot 6PM, the sum of reference internet bandwidth

is 20 (A) + 25 (B) = 45 which is more than the actual internet bandwidth. Therefore we need to use Knaster Sealed Bid algorithm to find who will take this time slot. A premise for this game is, from a player's perspective, all the time slots he required (reserved) at first are equally important. So, at time slot 6PM, A will bid 1/6 * 1000 = 167 and B will bid 1/6 * 1000 = 167. Here 6 means the number of remaining time slots the player wants. The bid from A and B are equal so we flip the coin and give the time slot to the one who win. In this case, we assume that A will win and get that time slot with full internet bandwidth (30). Then, A will need to pay for B 167 in order to get that time slot. A will have 833 and B will have 1167. We will apply the same procedure for the rest of the time slots.

A presentation of your results (graphs, data charts, whatever is best for making your point):

-Table below will summaries all information of our analysis on our project:

Knaster Sealed Bid table (for 2 people):

Time Slot	A	B	Who gets
9AM	10	2	A
10AM	10	2	A
6PM	6	1	A
9PM	2	8	B
10PM	2	8	B
11PM	2	8	B
Calculation			
$ Value of all time slots	32	29	
Fair Share	16	14.5	
$ Value of Time Slots won	26	24	
Difference (Fair Share - $ Value Won)	-10	-9.5	
Distribution of estate money pot of cash	9.75	9.75	
Final disposition of money	-0.25	0.25	

As we can see from table, A will need to pay B $.25 in order to use internet as those time slots which he needs.

Proportional Divider table (for 2 people):

Time Slot	A	B	SUM RATE	A's Proportional	B's Proportional	A's internet bandwidth	B's internet bandwidth
4PM	4	5	9	(4/9)	(5/9)	(4/9)*30 = 13.33Mbps	(5/9)*30 = 16.67Mbps
7PM	2	1	3	(2/3)	(1/3)	(2/3)*30 = 20Mbps	(1/3)*30 = 10Mbps
8PM	2	1	3	(2/3)	(1/3)	(2/3)*30 = 20Mbps	(1/3)*30 = 10Mbps

First approach results:

- A gets time slots [8AM, 9AM,10AM, 5PM,6PM]
- B gets time slots [11AM, 12AM, 2PM, 3PM, 9PM, 10PM, 11PM]
- Share 50/50 internet bandwidth time slots [12AM-7AM, 1PM]
- Share different internet bandwidth time slots [4PM, 7PM, 8PM]
- B also get pay from A $0.25

Second approach results:

- A gets time slots [8AM , 5PM, 6PM, 8PM, 10PM] with 30Mbps internet bandwidth
- B gets time slots [11AM, 12AM, 2PM, 3PM, 7PM , 9PM, 11PM] with 30Mbps internet bandwidth
- Share 50/50 internet bandwidth time slots [12AM-7AM, 1PM]
- Share different internet bandwidth time slots [9AM(A/10; B/10), 10AM(A/10; B/10), 4PM(A/20, B/10)]

Discussion and Conclusions based on the analysis and data:

- Discussion: Our first approach is initial approach where we first try to solve problem in project. However, our first approach seems only solve partial of problem. Therefore, we also work on second approach to get better results in fairness and envy-free situation. We will discuss and analysis in details for each approaches below.
    - From the result of the first approach, it shows that the internet is divided fairly between roommates. However, there is still envy between all players. On one hand, those time slots are not used by any one, the internet bandwidth in the house is divided 50/50 so there should be no envy between roommates and it should be proportional. On the other hand, the time slots which are needed by only one person are given to that person so there is no envy for the other person because he does not need to use internet during that time. For those time slots which are needed by one person but it might not needed by others, we used Knaster Sealed Bid to decide who will get time slots and how much that person needs to pay for the one who does not

use internet. In this case, it should be envy free for person who does not use it because he will get pay by others. Moreover, person who will use internet will also not envy because he can use full internet bandwidth for this tasks. For those time slots which are needed to use by everyone, we divide internet bandwidth which is based on the importance of tasks. The importance of each task is given by each player at the beginning of the process. In this case, it should base on the trust between players. If there is player who wants to take more time slots for his own use, he might lie about tasks and its importance. Then, internet bandwidth will be shared for him more than what he needs; it will create envy from other player. For most of the process of first approach, it should work well in different situation. The only concern is the last algorithm (proportional divider) which is applied in the process is depended in people's honesty. Therefore, we try to find other approach in order to eliminate the problem. In the end, we came up with the second approach which is "Optimized Knaster Sealed Bid Algorithm".

- In the second approach, internet is shared among players in a more precise and fair way.
    1. Precision: Because different internet service requires different bandwidth, players should tell the referee how much bandwidth they want at each time slot. Just as what we stated before, internet is a divisible item. With bandwidth requirement specified explicitly by each player, internet is divided more efficiently. This advantage is shown clearly in the "tricky" situation mentioned in the 3rd part of this paper.
    2. Fairness: When considering the fairness, we adopted idea from some process scheduling algorithms used by modern operating systems. At the first step of this algorithm, each player is given the same number of credits, which prevents the situation that "wealthy" players block "poor" players out of the game. As the algorithm goes on, credits are transferred among the players when bid occurs. The losers at a certain round should have more chance to win some rounds afterward. By doing so, an approximate fairness is guaranteed.

- Problems: there are several problems (coding and theory) which we still need to address in the future:
    - When we use both approaches to divide internet bandwidth for people in the same apartment, we require having outsider who has control over internet bandwidth in order to divide internet bandwidth fairly between people.

- The main point for first approach is that we need to collect all players' references for all time slots at the beginning of the process. However, we got the references from all players at the time of each bid in the program.
- At the beginning, we try to random each time slot in the program for each player from range 0 to 24. However, it will take many tests run to get both players have same time slot so we put down the range to 0-12. It gives more results to evaluate the algorithm.
- In the program for the first approach, each player only random 1 time slot every time of bidding. This is not realistic because there are tasks require more than 1 hour time slot. In the future, we need to develop the way to let player get more than 1 hour time slot at each bid.
- For the first approach, the Knaster Seal Bid algorithm requires to know all items and players' references at the begging of the process in order to calculate values and shares. However, we collect data at each time of bidding so Knaster Seal Bid does not fit very well in this program. For next developing test, we need to improve our program which will be more suitable with situations.
- On the other hand, we also need to find the better method of trading time slots if someone gives up.
- For second approach, we also need to find out better way to define how each player knows how much internet bandwidth they use for each task. If there is person who knows nothing about internet bandwidth, it will be big disadvantage.
- For both approaches, envy-free is not guaranteed but second approach has better results than first approach.
- To make the bidding policy simple, we made an assumption in the second algorithm. For one player, all the time slots he ordered are equally important, which is obviously not realistic. Although only bandwidth information is needed as input data, different bandwidth reservation means different types of internet service in our real life. So in the future, we need to design a more specific bidding policy.

- Conclusion: we applied three different algorithms in our first approach (moving knife, Knaster seal bid, and proportional divider) and extended algorithm of Knaster Seal Bid for second approach. For most of the cases, both approaches work well in the simple situations which are people doing not use any tricks in order to get the best result for themselves. Comparing between two approaches, second approach is more effective and efficient way to divide time slots between roommates. Second approach also gives reasonable share credits between players. In general, we believe that second approach will give better deal of properties dividing between all players.