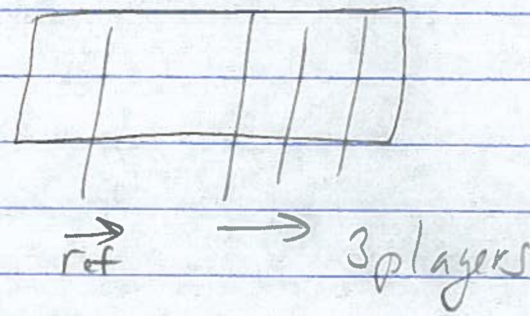
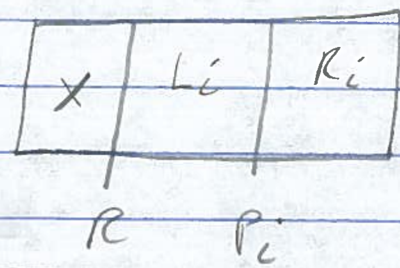
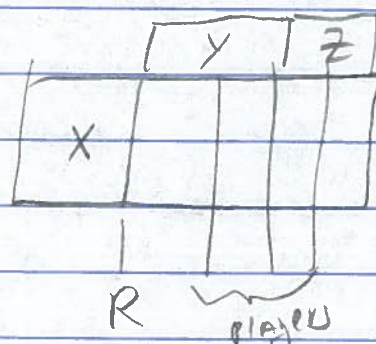
Stromquist moving knives
$n=3$ envy free



Rules
1) Ref moves knife to the right
2) Each player moves his/her knife to the right



$$\boxed{\text{STRATEGY: } \forall i \quad V_i(L_i) = V_i(R_i)}$$



3) Player X calls STOP, receives X & the middle player's knife also cuts, producing Y & Z

Say P_x gets X
Of the other 2 players, the one whose
knife is closer to the ref gets Y
The other player gets Z

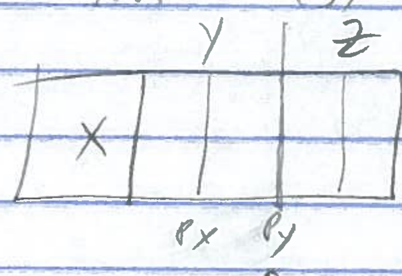
We will show this is envy-free
(& therefore proportional)

1) The player who stops P_x knows
P_x gets X & knows the
other 2 pieces are Y & Z
P_x cannot envy anybody – P_x is
choosing X over Y & Z

2) The other players must think
Y or Z is better than X
since they did not say STOP

P_y receives Y so either
P_x's knife is left of P_y's (a)
or NOT (b)

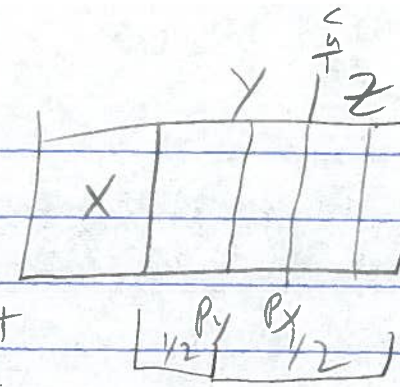
a)



↑ P_y was cutter

$v_y(Y) = v_y(Z)$
P_y gets Y & thinks
it's bigger than X

b)

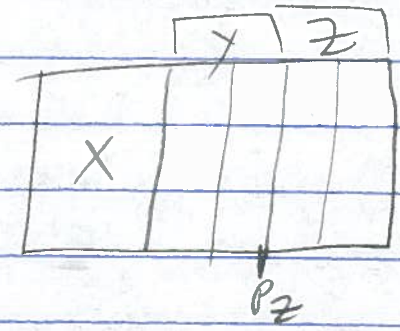


Value of rest of cake

$$V_y(Y) \geq V_y(Z) > V_y(X)$$

$\therefore$ P_y cannot envy

3) P_Z either was cutter or not

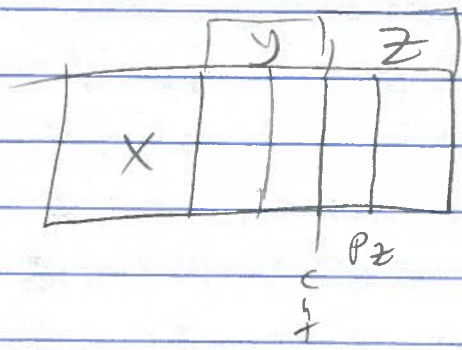


cutter

$$V_Z(Y) = V_Z(Z) > V_Z(X)$$

P_Z cannot envy

OR



$V_Z(Z) \geq V_Z(Y) > V_Z(X)$

No envy!