1.

9/24

CSE 544T

prop = proportional

Exam I, Oct 8 (monday)

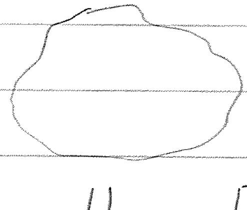
open note, closed internet. Look at * questions

So far:

* • Divide & choose, n = 2
• Dubins Spanier moving knife*, prop n people
• Austin moving knives, 2 parties decide "1/2"
  - 2 parties decide m "1/m"
  - 2 parties divide cake into m "equal" pieces

* • Branch-Knuster trimming *
  • Fink's lone chooser
    - n parties, dynamic
  • Woodall's strongly prop, n = 2
  • Steinhaus-Kuhn lone divider prop n = 3
  • Conway-Guy-Selfridge, n = 3 envy free

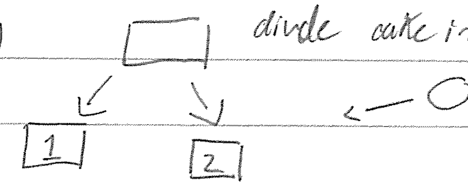
Fink lone chooser:



cake divided

new person joins, redistribute the cake proportionality

1) divide cake into 2.



new person

selects a percent piece from each.

2.

Fink's lone chooser

start $n=2$ [divide & choose]

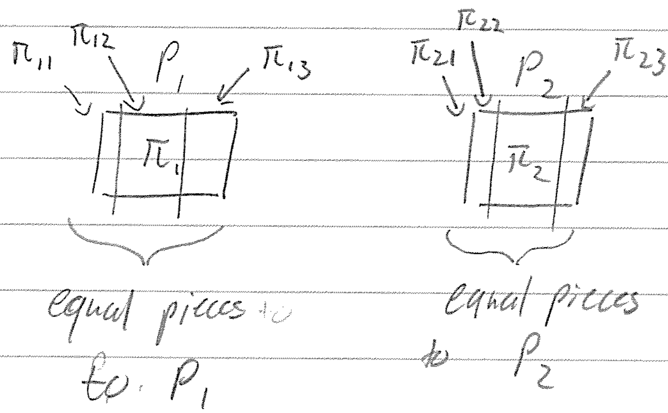
P_1 creates X_1, X_2 where $V(X_1) = V(X_2) = \frac{1}{2}$

Suppose P_2 chooses, takes π_2

P_1 gets π_1

standard divide & choose

Along comes P_3



P_1 $V(\pi_{11}) = V(\pi_{12}) = V(\pi_{13}) = (\frac{1}{2})(\frac{1}{3}) = \frac{1}{6}$ to P_1

P_2 $V(\pi_{21}) = V(\pi_{22}) = V(\pi_{23})$, each $\geq \frac{1}{6}$ to P_2

P_3 gets to pick one piece from π_1* and one piece from π_2*

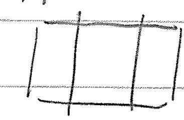
One piece from P_1 and one from P_2

P_3 takes one piece from P_1 so the value of the remaining cake $= \frac{2}{6} = \frac{1}{3}$, $n=3$ so this is proportional.

For P_2 it is the same value left to $P_2 \geq 2 * \frac{1}{6}$ or $\geq \frac{1}{3}$ $n = 3$, proportional.

What about P_3's value of the cake?

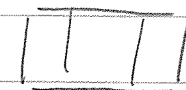
P_1's piece



π_1

value $P_3 = \alpha$

P_2's piece



π_2

value to $P_3 = 1 - \alpha$

If the whole piece of cake has value v, then

to the divider.

3 subpieces

Then not all 3 pieces can have value $< \frac{v}{3}$.

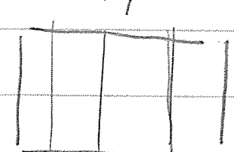
P_3 gets at least $\frac{1}{3}\alpha$ and $\frac{1}{3}\alpha - 1$

$$\frac{1}{3}\alpha + \frac{1}{3}(1-\alpha) = \frac{\alpha + 1 - \alpha}{3} = \frac{1}{3}$$

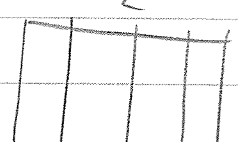
$\therefore$ P_3 picks up cake from P_1 & P_2 with a value of at least $\frac{1}{3}$ from P_3's perspective

Along comes P_4 divide cake into 4 pieces

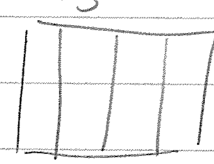
P_1



P_2



P_3



This is not envy free.

The number of cuts made $O(n^3)$ cuts!

$n=$		
2	1	
3	5	$= 4 + 1$
4	14	$= 9 + 4 + 1$

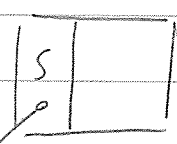
* Woodalls algerithm Strongly peporrional * $n=2$

related literature

- Problem of the nile, 1938 - 1961
- Ham Sandwich thoram, 1942 - 1985

The cakes value must differ to the two parties.

assume cake K



Diff value 2 people

$V_1(S) \neq V_2(S)$

& $V_1(S) > V_2(S)$

* From the above, show that it is possible from K to create $\boxed{X_1}$ $\boxed{X_2}$ where X_1 and X_2 are not necessarily the whole cake.

Such that

X_1 racks $V_1(X_1) > V_1(X_2)$

X_2 racks $V_2(X_1) < V_2(X_2)$

Problem

Given x_1, x_2, & the rest of the cake,

Assign cake π_1, π_2 such that:

$$V_1(\pi_1) > \frac{1}{2}$$

$$V_2(\pi_2) > \frac{1}{2}$$

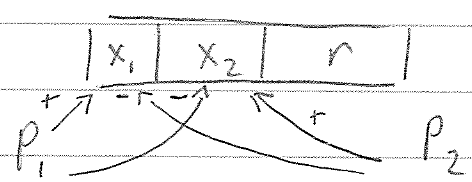
Already have P_1 likes x_1 and P_2 likes x_2 and the rest of the cake, r.

P_1 and P_2 ~~ssimply~~ play divide & choose on r, creating r_1 & r_2.

Analysis for person 1 (P_1)

Suppose $V_1(x_1) = \alpha_1$ $V_1(x_2) = \alpha_2$

$V_1(r) = 1 - \alpha_1 - \alpha_2$



By divide and choose P_1 gets r_1, P_2 gets r_2 and $V(r_1) \geq \frac{1 - \alpha_1 - \alpha_2}{2}$ for P_1 & P_2

Then P_1 takes x_1 and P_2 takes x_2.

P_1's value

$V(x_1) = \alpha_1$

$V(r_1) \geq \frac{1 - \alpha_1 - \alpha_2}{2}$

$V(x_1 \,\&\, r_1) \geq \alpha_1 + \frac{1 - \alpha_1 - \alpha_2}{2} \Rightarrow \frac{2\alpha_1 + 1 - \alpha_1 - \alpha_2}{2}$

$V(x_1 \,\&\, r_1) \geq \frac{1}{2} + \frac{\alpha_1 - \alpha_2}{2}$

$\therefore V > \frac{1}{2}$ because $\alpha_1 > \alpha_2$

Steinhaus - Kuhn

lone divider $\underset{\text{example}}{n=3}$, complicated

① P_1 creates X_1, X_2, X_3 where $V(X_1) = V(X_2) = V(X_3) = \frac{1}{3}$ from cake
② P_2 determines $S_2 \subseteq \{X_1, X_2, X_3\} \mid \forall x \in S_2 \; V_2(x) \geq \frac{1}{3}$
[lemma $|S_2| \geq 1$]
③ P_3 does the same thing to create S_3 using X_1, X_2, X_3

Two possibilities

- a) Lucky $|S_2| > 1$, Assign from smallest to largest $|S_2|$, $|S_3|$ size.

  Ex. $S_2 = \{X_1, X_3\}$ $S_3 = \{X_3\}$

  $\pi_3 = X_3$ $\pi_2 = X_1$ and person 1 gets whatever is left over because they like every piece equally.

- b) Unlucky $|S_2| = |S_3| = 1$

  There must be an unacceptable piece (say X_1) such that $V_2(X_1) < \frac{1}{3}$ $V_3(X_1) < \frac{1}{3}$, so give X_1 to P_1. Then combine X_2 & X_3 and play divide and choose with P_2 & P_3.

<u>Not envy free.</u> P_1 can envy in case b. (P_2 & P_3 divide & choose)
In case a P_2 can envy P_3 but not P_1. P_1 cannot envy ~~anyone~~ in a because he values each piece equally.