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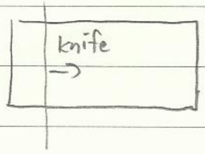
Divide & choose fails $n>2$

Dubins–Spanies (moving knife)



cake

Referee moves knife left-to-right

Rules

Any player can say stop at any time

That player gets piece to left of knife

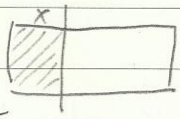
When one player remains he/she gets the rest of the cake

Otherwise, knife continues its toward left to right

Strategy

you must say STOP

Party i when



$V(x) = \frac{1}{n}$

why this strategy compelling?

Thm3 proof

Each party who called STOP so far see $1/n$ value

what about those who have not call STOP?

Failure to say STOP $\Rightarrow$ The piece on the left $< \frac{1}{n}$

Call the set of allocated pieces S, each element of S must be worth $< \frac{1}{n}$

Consider hungry people P

$V_P(S) \leq \underbrace{(n-k)}_{\text{\# allocated pieces}} \frac{1}{n}$

$V_P(\underset{\text{hole cake}}{K} - \underset{\text{allocated piece}}{S}) \geq \underbrace{1 - \frac{n-k}{n}}_{\text{value to } P \text{ of remaining cake}}$

$\geq \frac{K}{n}$

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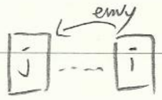
Thm 4

person i cannot envy j, $j \le i$

proof by contradiction

suppose $V_1(\pi_j) > \frac{1}{n}$



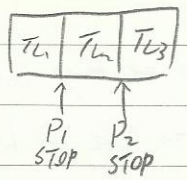
Then i would have said STOP but didn't

But i can envy j, $j > i$

$V_1(\pi_1) = \frac{1}{3}$

$V_1(\pi_2) > \frac{1}{3}$

$V_1(\pi_3) < \frac{1}{3}$

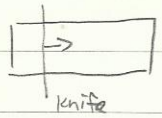


$V_2(\pi_2) = \frac{1}{3}$

Austin's Alg

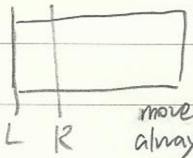
Setting 2 people to agree on what ½ cake is

start out as with Dubins Spanier



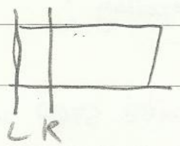
one of two people says STOP
when that party saw ½ value

Give P_1 second knife



move knifes to right
always keeps ½ value
between them

when P_2 see ½ value between knifes,
say STOP



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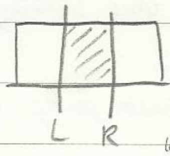
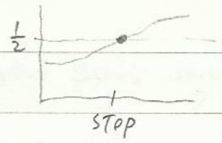


piece between
knifes has ½
value to both
people

Prove, P_2 eventually sees
$V_2(X) = \frac{1}{2}$ between knifes



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Austin's extension

A cake can be divided into a set S

$|S| = m$

such that

$\forall_{x \in S} \; V_1(x) = V_2(x) = \frac{1}{m}$

every piece has $\frac{1}{m}$ value to both parties

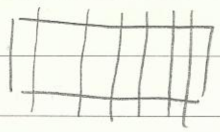
First

Two parties find 1 piece whose value is $1/m$ to both

Procedule :

Pl marks the cake, so as to delineate m pieces each worth $1/m$

example :
$m=7$



P_1 thinks each is worth $1/7$

Lemma

$\exists x \in S \mid V_2(x) \geq 1/m$

IF not, all pieces $< \frac{1}{m}$

breaks unit-value

Also $\exists x \in S \mid V_2(x) \leq \frac{1}{m}$

Maybe $\exists$ piece value $\frac{1}{m}$ – done

IF Not Lemma say $\exists x \; V_2(x) < \frac{1}{m}$

$\exists x \; V_2(x) > \frac{1}{m}$