Define

1) $\hat{\delta}(q, \lambda) = q$

2) $\hat{\delta}(q, za) = \delta(\hat{\delta}(q, z), a)$

Theorem

$$\hat{\delta}(q, xy) = \hat{\delta}(\hat{\delta}(q, x), y)$$

Cases a) $y = \lambda$

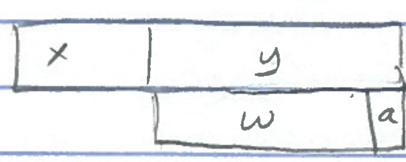
b) $y \neq \lambda$

CASE a) $y = \lambda$

	Why
$\hat{\delta}(q, xy) = \hat{\delta}(q, x\lambda)$	$y = \lambda$
$= \hat{\delta}(q, x)$	$x\lambda = x$
$= \hat{\delta}(\hat{\delta}(q, x), \lambda)$	Rule 1 Backwards
$= \hat{\delta}(\hat{\delta}(q, x), y)$	$y = \lambda$

$\square$

CASE b) $y \neq \lambda$ let $y = wa$, $w \in \Sigma^*$



$P(i) = |w| = i \rightarrow$ Theorem

Need to show $\forall i \geq 0 \quad P(i)$

$P(0)$: $\quad y = wa = \lambda a = a$

Show $\hat{\delta}(q, xa) = \hat{\delta}(\hat{\delta}(q,x), a)$

Lemma $\quad \forall a \in \Sigma \quad \hat{\delta}(q,a) = \delta(q,a)$

$\hat{\delta}(q,a) = \hat{\delta}(q, \lambda a) \qquad \lambda a = a$

$= \delta(\hat{\delta}(q,\lambda), a) \qquad$ Rule 2

$= \delta(q, a) \qquad$ Rule 1: $\hat{\delta}(q,\lambda) = q$

$\square$

Continuing $P(0)$

$\hat{\delta}(q, xy) = \hat{\delta}(q, xa) \qquad y = a$

$= \delta(\hat{\delta}(q,x), a) \qquad$ Rule 2

$= \hat{\delta}(\hat{\delta}(q,x), a) \qquad$ Lemma

$= \hat{\delta}(\hat{\delta}(q,x), y) \qquad y = a$

$\square$

Now show

$$\forall i \geq 0 \quad P(i) \rightarrow P(i+1)$$

$$P(i) \rightarrow \hat{\delta}(q, xy) = \hat{\delta}(q, xwa) \qquad y = wa$$

$$= \delta(\hat{\delta}(q, xw), a) \qquad \text{Rule 2}$$

$$= \delta(\hat{\delta}(\hat{\delta}(q, x), w), a)$$

The above is implied by $P(i)$

Because $|w| = i$,

$$\hat{\delta}(q, xw) = \hat{\delta}(\hat{\delta}(q, x), w)$$

$$= \hat{\delta}(\hat{\delta}(q, x), wa) \qquad \text{Rule 2 Backwards}$$

$$= \hat{\delta}(\hat{\delta}(q, x), y) \qquad y = wa$$

□