any of the elements of that set. For example, one of the new states is $\{1, 2, 5\}$; in the original FA, $\delta(1, a) = 8$, which tells us that the a-transition from $\{1, 2, 5\}$ goes to $\{3, 4, 8\}$. (If there are any inconsistencies, such as $\delta(5, a)$ not being an element of $\{3, 4, 8\}$, then we've made a mistake somewhere!)

EXERCISES

2.1. In each part below, draw an FA accepting the indicated language over $\{a, b\}$.

a. The language of all strings containing exactly two a's.

b. The language of all strings containing at least two a's.

c. The language of all strings that do not end with ab.

d. The language of all strings that begin or end with aa or bb.

e. The language of all strings not containing the substring aa.

f. The language of all strings in which the number of a's is even.

g. The language of all strings in which both the number of a's and the number of b's are even.

h. The language of all strings containing no more than one occurrence of the string aa. (The string aaa contains two occurrences of aa.)

i. The language of all strings in which every a (if there are any) is followed immediately by bb.

j. The language of all strings containing both bb and aba as substrings.

k. The language of all strings containing both aba and bab as substrings.

2.2. For each of the FAs pictured in Fig. 2.43, give a simple verbal description of the language it accepts.

2.3. a. Draw a transition diagram for an FA that accepts the string $abaa$ and no other strings.

b. For a string $x \in \{a, b\}^*$ with $|x| = n$, how many states are required for an FA accepting x and no other strings? For each of these states, describe the strings that cause the FA to be in that state.

c. For a string $x \in \{a, b\}^*$ with $|x| = n$, how many states are required for an FA accepting the language of all strings in $\{a, b\}^*$ that begin with x? For each of these states, describe the strings that cause the FA to be in that state.

2.4. Example 2.7 describes an FA accepting L_3, the set of strings in $\{0, 1\}^*$ that are binary representations of integers divisible by 3. Draw a transition diagram for an FA accepting L_5.

2.5. Suppose $M = (Q, \Sigma, q_0, A, \delta)$ is an FA, q is an element of Q, and x and y are strings in Σ^*. Using structural induction on y, prove the formula

$$\delta^*(q, xy) = \delta^*(\delta^*(q, x), y)$$