| **CSE 547T Intro. to Formal Languages and Automata** | **Spring 2013** |
|---|---|
| Exam II | |
| Given: 11 April 2013 | Due: End of class |

This exam is open-book and open-notes. **You may not use any electronic or online resources without explicit and prior permission from the professor.** Where possible, answer questions on the pages of the exam. Additional ruled pages are included at the end of the exam. Partial credit will be given where sufficient detail is provided. You must sign the pledge below for your exam to count. Any cheating will cause the students involved to receive an F for this course. Other unpleasant actions may also be taken.

| Name: | | |
|---|---|---|
| Student ID: | | |
| Problem Number | Possible Points | Received Points |
| 1 | 15 | |
| 2 | 10 | |
| 3 | 15 | |
| 4 | 25 | |
| 5 | 15 | |
| 6 | 20 | |
| 7 extra credit | 10 | |
| Total | 100 | |

**Pledge:** On my honor, I have neither given nor received any unauthorized aid on this exam.

Signed: ______________________________

The following languages (problems) can be used as attributed below for your solutions on this exam. If there is some other language or problem you wish to use, ask the instructor first.

| RE but not recursive | not RE (and thus also not recursive) |
|---|---|
| $\{\, e(T) \mid T \text{ halts on } \lambda \,\}$ | $\{\, e(T) \mid T \text{ does not halt on } \lambda \,\}$ |
| $\{\, e(T)e(w) \mid T \text{ halts on } w \,\}$ | $\{\, e(T)e(w) \mid T \text{ does not halt on } w \,\}$ |
| $\{\, e(T) \mid T \text{ accepts } e(T) \,\}$ | $\{\, e(T) \mid T \text{ does not accept } e(T) \,\}$ |
| $\{\, e(T)e(q) \mid T \text{ enters nonhalting state } q \,\}$ | $\{\, e(T)e(q) \mid T \text{ does not enter nonhalting state } q \,\}$ |
| $\{\, e(T) \mid L(T) \neq \emptyset \,\}$ | $\{\, e(T) \mid L(T) = \emptyset \,\}$ |
| | $\{\, e(T) \mid L(T) \text{ is recursive} \,\}$ |
| | $\{\, e(T) \mid L(T) \text{ is not recursive} \,\}$ |

1. **(15 points)** Answer the following (after some careful thought, but if you get stuck, move on and come back to these):

   (a) **(1 points)** If $L$ is a regular set, then its complement can be nonrecursive (true or false).__________

   (b) **(2 points)** If $L$ is a recursively enumerable (RE) language, then the strings in $L$ can be accepted by a __________.

   (c) **(2 points)** If $L$ is a recursively enumerable (RE) language, then the strings in $L$ can be generated by an __________ grammar.

   (d) **(2 points)** If $L$ is recursively enumerable (RE), then its complement must be RE (true or false). __________

   (e) **(2 points)** If $L$ is RE but not recursive, then its complement cannot be RE (true or false). __________

   (f) **(2 points)** The language of any old school TM whose $\delta$ table specifies only left moves is regular (true or false). __________

   (g) **(2 points)** Consider the alphabet $\Sigma = \{ a, b \}$. The number of languages with $\Sigma$ as their alphabet is (less than, equal to, more than) the number of Turing machines that could be constructed with $\Sigma$ as their alphabet (pick one).

   (h) **(2 points)** For every language $L$ over the alphabet $\Sigma = \{ a, b \}$, then there is an unrestricted grammar $G$ that can generate exactly $L$'s strings (true or false).

2. (10 points) In a homework problem you showed that a solution to the halting problem could resolve Fermat's last theorem:

$$(\forall X > 0)\ (\forall Y > 0)\ (\forall n > 2)\ \ \nexists Z > 0 \mid X^n + Y^n = Z^n$$

   (a) (2 points) Below, sketch the program you would show to a "halting-deciding eyeball" to resolve Fermat's last theorem.[1] Your program should ignore its input and establish:

   halting problem is decidable $\Longrightarrow$ Fermat's theorem resolved

   (b) (4 points) What does this result say on its own, if anything, about the decidability of the halting problem?

   (c) (4 points) What does this result say on its own, if anything, about the resolvability of Fermat's last theorem?

[1] This is simply the solution to the homework problem. I will sell you a copy of the program for 5 points.

3. **(15 points)** Consider the following (infinite) set that also appears on the cover page of this exam:

$$T = \{\, e(T) \mid T \text{ halts on } \lambda \,\}$$

As stated on the cover page, this set $T$ is RE but not recursive.[2]

(a) **(5 points)** Describe a process that creates an infinite *recursive* subset $S$ from $T$.

(b) **(3 points)** Prove (argue) that $S$ is infinite.

(c) **(3 points)** Prove (argue) that $S$ is recursive.

(d) **(4 points)** Since $S$ is recursive, describe a procedure that, given a TM $t$, could decide if $t \in S$ (or not).

[2] If you need more explanation about $T$ for the purposes of this problem, come forward and ask (no charge).

4. (25 points) Consider the nonrecursive set

$$P = \{\, e(T) \mid T \text{ eventually visits (enters) all but one of its nonhalting states} \,\}$$

In other words, consider some $e(T) \in P$. If $T$ has $n$ nonhalting states, $T$ at some point will have visited $n-1$ of them but it will never visit one of them. The never-visited state is not named explicitly in this problem.

> *We can also describe $P$ as the following problem: decide if a given Turing machine $T$ will eventually visit all but one of its nonhalting states.*

Without using Rice's theorem (because it does not apply here), show by reduction that $P$ is nonrecursive by completing the parts below.

(a) (1 points) From which nonrecursive problem $Q$ will you reduce any instance of $Q$ to some instance of $P$?

(b) (1 points) Below, circle the one arrow that correctly represents what you must show:

$$P \text{ recursive} \Longleftarrow \Longrightarrow Q \text{ recursive}$$

(c) (8 points) Below, draw the boxes that illustrate your implication. Use the name "$T_{obj}$" for the TM that you prepare to show to the "eyeball".

*Continued on next page...*

(d) **(10 points)** Describe completely the behavior of the machine $T_{obj}$ that is inspected by the eyeball. How does this behavior establish the above implication? Explain why one problem's (yes or no) solution yields the (yes or no) solution to the other problem.

(e) **(5 points)** Based on the above, what allows you to conclude that $P$ is nonrecursive?

5. (15 points) Consider the language $L$. We write that language $L^R$ is the *reverse* of $L$ if

$$w \in L \Longleftrightarrow Rev(w) \in L^R.$$

That is, the strings in $L^R$ are exactly the reverse of the strings in $L$. For example, if $L = \{ cat, dog, dad \}$, then $L^R = \{ tac, god, dad \}$.

Both of the itemized statements below are true. Pick *one* of the statements and prove that that it is true:[3]

- If $L$ is a recursively enumerable set then so is $L^R$.
- If $L$ is a recursive set then so is $L^R$.

[3] A hint is available for 5 points.

6. (20 points) Pick any *two* of the problems below and prove that they are undecidable.

- If Rice's theorem (as covered in class) applies, you are free to use it, but you must explain why Rice's theorem applies. You will not receive credit for a problem if you use any other variant of Rice's theorem or if you use Rice's theorem where it does not apply.
- If you are writing a reduction, state the implication and draw the eyeball and boxes to receive partial credit.

Below are the three problems from which you should choose two. Each is worth 10 points.

- Recalling problem 5, consider the set

  $$S = \{\, e(T_1)e(T_2) \mid L(T_2) = L^R(T_1) \,\}$$

  That is, $S$ is the set of pairs of Turing machines, such that the language of one TM is the *reverse* (string-by-string) of the other TM's language.[4]
- $\{\, e(T) \mid L(T) \text{ contains at least two strings} \,\}$
- $\{\, e(T1)e(T2)e(T3) \mid L(T1) = L(T2) \cap L(T3) \,\}$[5]

[4] A hint is available for 5 points.
[5] A hint is available for 5 points.

7. Extra credit (10 points) The exam is scored out of 100 points, but with this extra-credit problem, it is possible to score 110 points. If you solve this problem correctly, the 10 points from this problem will be added to your exam score for the other problems. That total, scored out of 100, is your grade on this exam.

   Consider a *skipping* TM that always moves *two* squares to the right or left on each transition. Using bisimulation, show that the skipping TM is equivalent in power to the TMs discussed in class.