CSE 547T Intro. to Formal Languages and Automata	**Spring 2013**
Exam III	
Given: 7 May 2013	Due: 9 PM

This exam is open-book and open-notes. **You may not use any electronic or online resources without explicit and prior permission from the professor.** Where possible, answer questions on the pages of the exam. Additional ruled pages are included at the end of the exam. Partial credit will be given where sufficient detail is provided. You must sign the pledge below for your exam to count. Any cheating will cause the students involved to receive an F for this course. Other unpleasant actions may also be taken. If you filled out the course evaluation by the time of your taking this exam, you will be credited 5 points extra credit on Exam III.

Name:		
Student ID:		
Exam III		
Problem Number	Possible Points	Received Points
1	20	
2	20	
3	20	
4	40	
Total	100	

Pledge: On my honor, I have neither given nor received any unauthorized aid on this exam; moreover, I have read this cover page and understand what is required of me.

Signed: ______________________________

Important: For a proof involving reduction, **show the boxes and eyeball(s)** as you have been taught for this course. For a proof involving induction, **state the induction hypothesis** or you will receive no credit for your proof.

1. (20 points) Answer the following (after some careful thought, but if you get stuck, move on and come back to these):

   (a) (2 points) The strings of a context free language can be characterized by a __________ grammar and recognized by a __________ automaton.

   (b) (3 points) Given the specification of a PDA, it is decidable whether there is some string recognized by that PDA (true or false).__________

   (c) (3 points) Any CFG with exactly one production describes a regular language (true or false).__________

   (d) (4 points) Consider the following grammar:

$$\begin{array}{rcl} \mathsf{A} & \rightarrow & \mathsf{A\ a} \\ & | & \mathsf{A\ b} \end{array}$$

   Describe (using English if necessary) the language associated with this grammar:

   (e) (8 points) If G is a context-free grammar, it is decidable whether (answer each true or false):

      i. $L(G) = \emptyset$ __________

      ii. $L(G)$ is finite __________

      iii. $L(G)$ is infinite __________

      iv. $L(G) = \Sigma^\star$ __________

2. (20 points) Prove by reduction from PCP that the following problem is undecidable:

   Given two CFGs, G_1 and G_2, do they have 2 or more strings in common? In other words, is $|L(G_1) \cap L(G_2)| \geq 2$?

   In class and in our text, we studied a technique for generating grammars from an instance of PCP (in class, these grammars had start symbols S_A and S_B). To save time, there is no need to replicate that construction here; you may reference it and use it as you wish.

3. (20 points) Chose **one** of the following problems:

- Prove by reduction from any undecidable problem[1] we have discussed in class that the following problem is undecidable for context-free grammars G_1 and G_2:[2]

$$\textit{Is } L(G_1) \subseteq L(G_2)\textit{?}$$

- Consider the problem of **3-PCP**, which is like classic PCP, except that each tile has a top, middle, and bottom part. An instance of 3-PCP has a solution if there is an arrangement of dominos such that the messages on the top, middle, and bottom all agree. Prove by reduction that 3-PCP is nonrecursive.[3]

[1] If you are uncertain whether your source problem for the reduction has been covered in class, come forward and ask.

[2] A hint for this problem is for sale for 5 points.

[3] A hint for this problem is for sale for 5 points.

4. (40 points) Recall (from Exam I) that we can express an addition problem as a string. Consider encoding such a problem as follows:

$$L = \{ a^n b^m c^{n+m}, n \geq 0, m \geq 0 \}$$

In other words, the string has some number of as and bs, and the number of cs is the sum of the counts of the other two letters. For example, L includes the following strings: $\{ \lambda, abcc, aacc, aaabbbbccccccc \}$.[4]

It turns out that L is a context-free language.

(a) (5 points) Describe a PDA that recognizes strings in L. You do *not* need to specify a transition table: just describe how the PDA would work using prose.

(b) (5 points) Consider the following CFG G:

S	→	a S c
	|	T
T	→	b T c
	|	λ

Complete **one** of the following:

- (5 points) Below, draw a derivation tree for the string `abbccc`:

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[4]If you have any questions about this language, please come forward and ask!

- (5 points) Below, show a leftmost derivation of the string `abbccc`:

(c) (5 points) Is this grammar ambiguous? __________

(d) (5 points) Consider the string $z = a^n b^n c^{2n}$ which is clearly in L, and $|z| > n$, where n is the pumping-lemma constant. I will now try to "prove" L is not context free after all, by "applying" the pumping lemma as follows: Take $vwx, 1 \leq |vwx| \leq n$ as occurring exclusively in either the as, bs, or cs. In each of those 3 cases, consider v^0wx^0, which removes as, bs, or cs, respectively. In each case, the number of cs no longer equals the sum of the as and bs, so $v^0wx^0 \notin L$, thus "proving" that L is not context free.

What is wrong my "proof"? For full credit, provide a complete explanation of why the pumping lemma fails to prove L is not context free.

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(e) **(10 points)** Using induction, prove that $L(G) \subseteq L$:[5]

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[5] An induction hypothesis for this proof is for sale for 5 points.

(f) **(10 points)** Using induction, prove that $L \subseteq L(G)$:[6]

[6] An induction hypothesis for this proof is for sale for 5 points.