

HW 4 Solutions

2.22 (a)

$$L = \{ a^n b a^{2n}, n \geq 0 \}$$

$$z = uvw = a^n b a^{2n} \quad \text{so} \quad uv = a^k$$

$$\text{so} \quad v = a^j \quad 1 \leq j \leq n$$

$$\rightarrow uv^2w = a^{n+j} b a^{2n}, \quad 1 \leq j \leq n$$

$$\text{then } uv^2w \notin L \quad \square$$

(f)

$$L = \{ x \mid \text{no prefix of } x \text{ has} \\ \text{more } b\text{'s than } a\text{'s} \}$$

$$z = uvw = a^n b^n \in L \quad (\text{barely!})$$

$$\text{so } v = a^j$$

$$\rightarrow uv^0w = a^{n-j} b^n \quad 1 \leq j \leq n$$

$$\rightarrow \text{the prefix } a^{n-j} b^n$$

$$\text{has more } b\text{'s than } a\text{'s} \quad \square$$

$$2.59 \quad L = \{ a, b \} \cup \{ a^n b^n, n \geq 0 \}$$

Pumping lemma shows L not regular

but $L^* \subseteq \{ a, b \}^* = \Sigma^*$ is regular

$$2.22 (h) \quad L = \{ xx, x \in \Sigma^* \}$$

$$z = uvw = a^n b a^n b \quad \text{so } v = a^j \quad 1 \leq j \leq n$$

$$uv^2w = a^{n+j} b a^n b \notin L$$