<u>Theorem</u> Given NFA M uses λ, construct λ-free N

N accepts w if & only if

M accepts w

"if" part:

By induction on length of w

$Ih(n) = \left(N \text{ accepts } w \Leftrightarrow M \text{ accepts } w \right) \Leftarrow |w| = n$

Basis: $|w| = 0$

M accepts λ $\Leftrightarrow$ $\lambda\text{-close}(q_0, \lambda) \cap F_M \neq \emptyset$

$\Rightarrow$ $q_0 \in F_N$ by construction

Otherwise ... $q_0 \notin F_N$

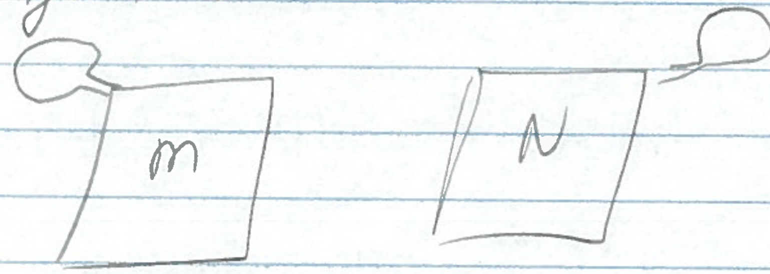
<u>Induction:</u> Assume M & N agree on

w, $|w| = n-1$

What about wa, $|wa| = n$?

We're stuck —

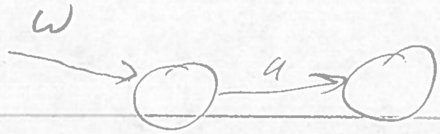
just because



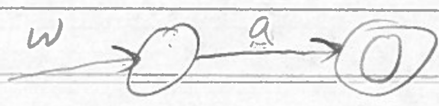
lights agree on w,

doesn't mean same behavior in

the future! (on wa)



Won't accept wa



Will accept wa

To prove observable equivalence,
we'll have to "look inside"

Make previous theorem a lemma for $0 = |\lambda| = |w|$

<u>Lemma</u> drop hats after here

$$\hat{\delta}_m(q, w) = \hat{\delta}_N(q, w) \quad \forall q \in Q,\ w \in \Sigma^*$$

IH(n): $|w| = n \rightarrow \delta_m(q, w) = \delta_N(q, w)$

may have λ — No λ, from construction

Basis $|w| = 0$

$$\forall q \in Q, \quad \delta_N(q, \lambda) = \{q\}$$

But $\quad \delta_m(q, \lambda) = \lambda\text{-close}(q, \lambda)$

Fails for $|w| = 0$
(But doesn't matter since we showed observational equivalence at $|w| = 0$)

ADD condition to this Lemma: $|w| > 0$

BASE CASE $|w| = \underline{1}$

For any $a \in \Sigma$,

$$a = \lambda^* a \lambda^*$$

and so $\delta_N(q, a) = \delta_m(q, a)$
by construction

<u>Induction</u>

show

$$\delta_M(q, w) = \delta_N(q, w) \leftarrow |w| = n-1$$

$$\Longrightarrow$$

$$\delta_M(q, wa) = \delta_N(q, wa) \leftarrow |wa| = n$$

$$\hat{\delta}_M(q, wa) = \delta_M(\hat{\delta}_M(q, w), a) \quad (\text{def } \hat{\delta})$$

$$= \delta_M(\hat{\delta}_N(q, w), a) \quad \text{IH}$$

$$= \delta_M^*(\hat{\delta}_N(q, w)\lambda^*, \lambda^* a \lambda^*) \quad [a = \lambda^* a \lambda^*]$$

$$= \delta_N(\hat{\delta}_N(q, w), a) \quad \text{BY CONSTRUCTION}$$

$$= \hat{\delta}_N(q, wa) \quad (\text{def } \hat{\delta})$$

□

<u>Now</u> try theorem again

Two cases:

1) M accepts λ $\quad \lambda\text{-close}(q_0, \lambda) \cap F_M \neq \emptyset$
   By previous lemma
2) M doesn't accept λ
   then $F_N = F_M$ by construction
   and $\hat{\delta}_N(q_0, w) = \hat{\delta}_M(q_0, w)$ by lemma

□