Please write all solutions clearly and legibly in the space provided.
For any NP-completeness proof you can only use the following problems for your reduction: CIRCUIT-SAT, FORMULA-SAT, 3-CNF-SAT, CLIQUE, VERTEX-COVER, SUBSET-SUM, PARTITION, HAM-CYCLE, TSP.

1. In the bin packing problem, the input is a set of $n$ objects, where the size $s_i$ of the $i$th object satisfies $0 < s_i \leq 1$. The goal is to pack all the objects into the minimum number of unit-size bins. (So each bin can hold any subset of objects whose total size does not exceed 1.)

   (a) (5 pts) State a decision version of the bin packing problem.

   (b) (20 pts) Prove that the decision version of the bin packing problem is NP-complete.
(c) (20 pts) The first-fit heuristic takes each object in turn and places it into the first bin that can accommodate it, starting a new bin as needed. Your goal is to prove a ratio bound of 2 for the first-fit heuristic. Below are some useful things to think about.

- Let $S = \sum_{i=1}^{n} s_i$.
- Would it be possible for the first-fit heuristic to leave more than one bin less than half full?
2. (25 pts) You are to prove that the following graph problem (GP) is NP-hard. The input consists of a directed graph $G = (V, E)$, a specified source vertex $s \in V$, a specified destination vertex $t \in V$, and a set $C = \{(a_1, b_1), (a_2, b_2), \ldots, (a_k, b_k)\}$ of pairs of vertices from $V$. The question is whether or not there is a directed path from $s$ to $t$ in $G$ that contains at most one vertex in each pair in $C$.

Observe that this path does not contain both vertices in any of the pairs in $C$.

$$C = \{ (a, f), (a, g), (b, j), (b, g), (d, k), (e, j) \}$$

For this example there is a solution as illustrated by the directed path:

$$s \rightarrow c \rightarrow e \rightarrow i \rightarrow k \rightarrow t$$

Observe that this path does not contain both vertices in any of the pairs in $C$. 
More Space for Problem 2.

<table>
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<tr>
<th>Problem</th>
<th>Points Possible</th>
<th>Points Received</th>
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