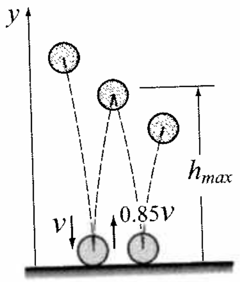
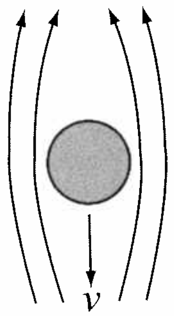
3. A ball that is dropped on the floor bounces back up many times, reaching a lower height after each bounce. When the ball impacts the floor its rebound velocity is 0.85 times the impact velocity. The velocity $v$ that a ball hits the floor after being dropped from a height $h$ is given by $v = \sqrt{2gh}$, where $g = 9.81\ \mathrm{m/s^2}$ is the acceleration of the Earth. The maximum height $h_{max}$ that a ball reaches is given by $h_{max} = \frac{v^2}{2g}$, where $v$ is the upward velocity after impact. Consider a ball that is dropped from a height of 2 m. Determine the height the ball reaches after the first 8 bounces. (Calculate the velocity of the ball when it hit the floor for the first time. Derive a formula for $h_{max}$ as a function of the bounce number. Then create a vector $n = 1, 2, \ldots, 8$ and use the formula (use element-by element operations) to calculate a vector with the values of $h_{max}$ for each $n$.)




4. If a basketball is dropped down from a helicopter, its velocity as a function of time $v(t)$ can be modeled by the equation:

$$v(t) = \sqrt{\frac{2mg}{\rho A C_d}}\left(1 - e^{-\sqrt{\frac{\rho g C_d A}{2m}}\,t}\right)$$

where $g = 9.81\ \mathrm{m/s^2}$ is the gravitation of the Earth, $C_d = 0.5$ is the drag coefficient, $\rho = 1.2\ \mathrm{kg/m^3}$ is the density of air, $m = 0.624\ \mathrm{kg}$ is the mass of the basketball, and $A = \pi r^2$ is the projected area of the ball ($r = 0.117\ \mathrm{m}$ is the radius). Determine the velocity of the basketball for $t$ = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 s. Note that initially the velocity increases rapidly, but then due to the resistance of the air, the velocity increases more gradually. Eventually the velocity approaches a limit that is called the terminal velocity.




5. The length $|\mathbf{u}|$ (magnitude) of a vector $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is given by $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$. Given the vector $\mathbf{u} = 14\mathbf{i} + 25\mathbf{j} - 10\mathbf{k}$, determine its length two ways:

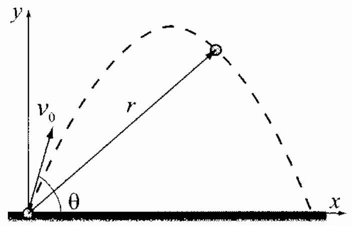
*a)* Define the vector in MATLAB, and then write a mathematical expression that uses the components of the vector.

*b)* Define the vector in MATLAB, then use element-by element operation to create a new vector with elements that are the square of the original vector. Then use MATLAB built-in functions `sum` and `sqrt` to calculate the length. All of these can be written in one command.



6. The position as a function of time $(x(t), y(t))$ of a projectile fired with a speed of $v_0$ at an angle $\theta$ is given by:

$$x(t) = v_0 \cos\theta \cdot t \qquad y(t) = v_0 \sin\theta \cdot t - \frac{1}{2}gt^2$$




where $g = 9.81\ \mathrm{m/s^2}$ is the gravitation of the Earth. The distance $r$ to the projectile at time $t$ can be calculated by $r(t) = \sqrt{x(t)^2 + y(t)^2}$. Consider the case where $v_0 = 100\ \mathrm{m/s}$ and $\theta = 79°$. Determine the distance $r$ to the projectile for $t = 0, 2, 4, \ldots, 20\ \mathrm{s}$.

7. Two vectors are given:

$\mathbf{u} = 4\mathbf{i} + 9\mathbf{j} - 5\mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + 6\mathbf{j} - 7\mathbf{k}$

Use MATLAB to calculate the dot product $\mathbf{u} \cdot \mathbf{v}$ of the vectors in two ways:

a) Define **u** as a row vector and **v** as a column vector, and then use matrix multiplication.

*b)* Use MATLAB built-in function `dot`.

8. Define $x$ and $y$ as the vectors $x$ = 2, 4, 6, 8, 10 and $y$ = 3, 6, 9, 12, 15. Then use them in the following expression to calculate $z$ using element-by-element calculations.

$$z = \left(\frac{y}{x}\right)^2 + (x + y)^{\left(\frac{y-x}{x}\right)}$$

9. Define $h$ and $k$ as scalars, $h = 0.7$, and $k = 8.85$, and $x$, $y$ and $z$ as the vectors $x = [1, 2, 3, 4, 5]$, $y = [2.1, 2.0, 1.9, 1.8, 1.7]$, and $z = [2.0, 2.5, 3.0, 3.5, 4.0]$. Then use these variables to calculate $G$ using element-by-element calculations for the vectors.

$$G = \frac{hx + ky}{(x + y)^h} + \frac{e^{\left(\frac{hy}{z}\right)}}{z^{(y/x)}}$$

10. Show that $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

Do this by first creating a vector $x$ that has the elements: 1 0.5 0.1 0.01 0.001 0.00001 and 0.0000001. Then, create a new vector $y$ in which each element is determined from the elements of $x$ by $\frac{e^x - 1}{x}$. Compare the elements of $y$ with the value 1 (use format long to display the numbers.