**Introduction**

**Shape anisotropy**

Objects in nature often contain parts that exhibit anisotropic elongation, such as tubes (1-D elongation) and plates (2-D elongation). These parts are often strongly associated with the structure, and/or function of the objects.

**Goal**

Compute a shape descriptor that captures object parts with different shape anisotropy.

Approach: compute a mixed-dimensional skeleton that consists of medial elements whose dimension reflects the dimension of anisotropic elongation:
- **Medial curve**: tube-like parts
- **Medial surface**: plate-like parts

Applications:
- Recognition
- Segmentation
- Comparison
- Retrieval
- Visualization

**Challenges**

Existing skeletonization works are dimension-specific (e.g., either surface or curve skeleton). Computing a skeleton with elements at multiple dimensions faces two challenges:
- **Defining medial geometry at lower-dimensions** (e.g., k-D medial geometry for N-D objects for k<N)
- **Identifying salient medial geometry at each dimension** (e.g., capturing shape anisotropy at that dimension)

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**Continuous analogy**

**Geodesic Grassfire**: An erosion that travels geodesically on a manifold from its boundary.

- Lower-dimensional manifolds form as the erosion fronts meet (e.g., dots in t2).
- A erosion front ends if the boundary vanishes (e.g., in t4)
- Erosion ends when there is no boundary left (e.g., t5).

**Defining medial geometry**: The k-D medial geometry of an N-D object is the set of k-D manifold formed during geodesic grassfire. (When k=N-1, it is the Medial Axes)

**Medial salience**: A point p on a k-D medial geometry is salient, if the erosion on the k-D manifold reaches p much later than the erosion on the (k+1)-D manifold. (Intuitively, the erosion time on a k-D manifold expresses the isotropic elongation of shape in k dimensions at p)

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**Discrete approximation**

**Geodesic Grassfire Thinning**: Iterative removal of boundary elements on a cell complex.

- Each iteration simultaneously deletes all pairs of cells on the boundary of the complex.
- Homotopy-preserving

**Medial geometry and salience**: The k-D medial geometry consists of isolated k-D elements during thinning.

- The salience of a medial k-D element is how long it stays isolated before it is removed.

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**The algorithm**

Computes a topology-preserving skeleton that captures shape anisotropy.

**Results**

**Examples**

**Applications**

The mixed-dimensional skeleton computed on a 3D protein model (BTV, left) distinguishes beta-sheets (plate-like, blue on the right) from alpha-helices and loops (tube like).

**Contributions**

A continuous erosion analogy:
- Define medial geometry in lower-dimensions
- Identify salient medial geometry at each dimension

A discrete algorithm:
- Computes a discrete skeleton that naturally captures shape anisotropy
- Simple to implement
- Generally applicable in any dimensions
- Robust to noise and discretization levels.

**Conclusion**