

Exam 2 (Redo)

December 11, 2003

Please write all solutions clearly and legibly in the space provided.

For any NP-completeness proof you can only use the following problems for your reduction: CIRCUIT-SAT, FORMULA-SAT, 3-CNF-SAT, CLIQUE, VERTEX-COVER, SUBSET-SUM, PARTITION, HAM-CYCLE, TSP.

1. In the **bin packing** problem, the input is a set of n objects, where the size s_i of the i th object satisfies $0 < s_i \leq 1$. The goal is to pack all the objects into the minimum number of unit-size bins. (So each bin can hold any subset of objects whose total size does not exceed 1.)

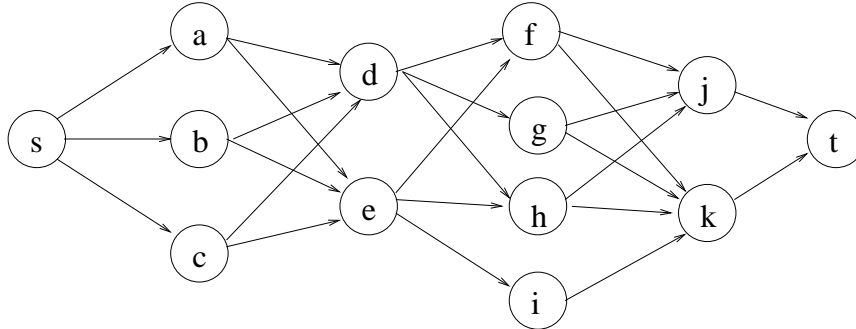
(a) (5 pts) State a decision version of the bin packing problem.

(b) (20 pts) Prove that the decision version of the bin packing problem is NP-complete.

(c) (20 pts) The **first-fit** heuristic takes each object in turn and places it into the first bin that can accommodate it, starting a new bin as needed. Your goal is to prove a ratio bound of 2 for the first-fit heuristic. Below are some useful things to think about.

- Let $S = \sum_{i=1}^n s_i$.
- Would it be possible for the first-fit heuristic to leave more than one bin less than half full?

2. (25 pts) You are to prove that the following graph problem (GP) is NP-hard. The input consists of a directed graph $G = (V, E)$, a specified source vertex $s \in V$, a specified destination vertex $t \in V$, and a set $C = \{(a_1, b_1), (a_2, b_2), \dots, (a_k, b_k)\}$ of pairs of vertices from V . The question is whether or not there is a directed path from s to t in G that contains at most one vertex in each pair in C .



$C = \{ (a,f), (a,g), (b,j), (b,g), (d,k), (e,j) \}$

For this example there is a solution as illustrated by the directed path:

$s \rightarrow c \rightarrow e \rightarrow i \rightarrow k \rightarrow t$

Observe that this path does not contain both vertices in any of the pairs in C .

More Space for Problem 2.

Problem	Points Possible	Points Received
1a,b	25	
1c	20	
2	25	
total	70	