CSE 241 Algorithms and Data Structures

Practice Problems for Homework 3

1. Prove the best lower bound you can (using the decision tree technique) on the number of distance computations needed to find the closest pair of n points in the plane under the model of computation in which you cannot directly access the coordinates of a point but instead can just compute the distance between two points.

2. Prove the best lower bound you can (using the decision tree technique) on the time complexity of a comparison based algorithm for the following problem: You are given a sorted array A (of n elements) and two elements x and y where $x \leq y$. The algorithm is required to compute how many elements in A are less than both x and y, how many elements of A are between x and y (inclusive), and how many elements of A are bigger than both x and y. Note that x and y are not necessarily in A.

3. Suppose you are given the task to sort one thousand 32-bit keys. You have decided to use radix sort for this problem and want to decide how many bits each radix sort digit. Which is best among having 1 bit per radix sort digit, 4 bits per radix sort digit, 8 bits per radix sort digit or 16 bits per radix sort digit? You are provided with a counting sort procedure with exact time complexity of $5n + 4k$. Show your work.

4. Give the asymptotically fastest algorithm you can to sort n integers in the range of 0 to $(n^4) - 1$. You should give a very clear and complete high-level description of your algorithm. Be sure to analyze the time complexity of your algorithm as a function of n. You are NOT restricted to use a comparison sorting algorithm (although are welcome to if you want).