

Final Exam

December 18, 2003

1. (15 points) Short Answer Questions

(a) (5 points) What would be the effect of modifying genetic algorithms to skip the step that uses the crossover mutation. Give a two to three sentence explanation for your answer.

(b) (5 points) Suppose you have a NP-complete problem for which you want to develop a c -approximation algorithm using LP-relaxation. Carefully describe the steps you must take to do this. Be as explicit as you can.

(c) (5 points) Consider a new problem called XYZ. Suppose you prove XYZ is in NP and then prove that $XYZ \leq_P \text{Vertex Cover}$. What conclusion(s) follow from these two facts and the fact the Vertex Cover is NP-complete?

2. (25 points) Consider the following problem. You are given two inputs, an integer k and an array A containing n integers. The problem is to find *any one* of the k smallest elements of A . In other words, your algorithm must return one of the k smallest elements of A but it doesn't matter which one. Give the best lower bound (as a function of n and k) you can for any comparison based algorithm to solve this problem.

You should spend no more than 20 minutes thinking about the best lower bound you can prove. Then write it up and carefully prove whatever bound you claim. Then only return to try and prove a better lower bound if you have time at the end. *It is more important that you carefully prove the bound you give than giving a better bound but without adequate proof.*

3. (25 points) Here we return to the rent-to-buy problem. Let the cost of buying a pair of skis be 20 times more than the rental cost. Give a randomized on-line algorithm A for which the expected amount of money that A payed to rent and/or buy skis satisfies

$$\text{For all inputs } \sigma, \quad E[C_A(\sigma)] \leq c \cdot \text{opt} + a$$

for $c < 2$ and a any constant where opt is the amount of money used by the optimal solution, and $E[C_A(\sigma)]$ is the expected cost of A for input sequence σ .

Be sure that clearly describe your algorithm A , and prove that your stated upperbound for $E[C_A]$ holds. For this exam, there is no need to see how small you can make c . Once you have a solution with $c < 2$ (and have proven it), move on.

4. (35 points) For the following problem either give the most efficient polynomial time algorithm you can to solve it or prove that the decision version is NP-complete. If you give an algorithm, be sure to prove that your algorithm is correct and analyze the asymptotic time complexity. If you prove that the decision version is NP-complete, be sure to clearly define the decision problem and also carefully prove that your reduction is correct.

For a 5 point hint, I will tell you if you should give a greedy algorithm, use dynamic programming, or prove it is NP-complete.

Here is the problem: You are given as input an n row by m column matrix of bits. The goal is to find the area of the largest square submatrix that contains all 1s. For example, for the below input, the answer is 9 for the 3 by 3 square of 1s.

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1 1 1 0 0 0 0
1 1 1 1 0 1 1
0 1 1 1 0 0 1
0 1 1 1 1 0 1
0 0 1 0 0 1 0
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More Space for Problem 4.

I had a really nice problem in which you were to develop an approximation algorithm using LP-relaxation but after some thought I decided you'll have had enough without it :-)
If you want to see the problem, I'd be happy to send it to you.

Extra Space for Problem 2 or 3 (if needed). Clearly indicate which problem it is for.

Problem	Points Possible	Points Received
1	15	
2	25	
3	25	
4	35	
total	100	