1. Answer each of the following questions and briefly explain your answer. You are given as facts that 3-CNF-SAT and TSP (traveling salesman problem) are NP-complete and that there is a polynomial time algorithm for 2-CNF-SAT.

   (a) True or false: 3-CNF-SAT $\leq_p$ TSP.
   (b) True or false: If $P \neq NP$, then no NP-complete problem can be solved in polynomial time.
   (c) True or false: 3-CNF-SAT $\leq_p$ 2-CNF-SAT (assume that $P \neq NP$).

2. A feedback vertex set in a directed graph $G = (V, E)$ is a subset $V'$ of $V$ such that $V'$ contains at least one vertex from each directed cycle in $G$. The feedback vertex set problem (FVS) is: Given a directed graph $G$ and an integer $\ell$, does $G$ have a feedback vertex set with at most $\ell$ vertices? Prove that VERTEX-COVER $\leq_p$ FVS. Then answer the below question.

   Does it follow from the fact that VERTEX-COVER $\leq_p$ FVS that FVS is NP-complete? Why?

3. The Hamilton cycle problem (HAM-CYCLE) that we have studied is defined for undirected graphs. We now consider the directed Hamilton cycle problem (DIRECTED-HAM-CYCLE) in which you are given a directed graph $G$ and asked if there is a directed cycle that visits every vertex exactly once. In this problem we consider a proposed reduction to show DIRECTED-HAM-CYCLE $\leq_p$ HAM-CYCLE. Here is the proposed input transformation function $f$. Let $G = (V, E)$ be the directed graph that is input for DIRECTED-HAM-CYCLE. We define the undirected graph $G' = (V', E')$ for HAM-CYCLE as follows. For each vertex $v \in V$ there are two vertices $v_{in}$ and $v_{out}$ placed in $V'$. $E'$ contains $|V| + |E|$ edges. First, for each $v \in V$ an edge is placed between $v_{in}$ and $v_{out}$. Then for a directed edge from $u$ to $v$ in $E$, we place an undirected edge from $u_{out}$ to $v_{in}$ in $E'$. Here is an example to make sure that the definition for $G'$ is clear.

   Prove or Disprove: If $G$ has a directed Hamilton cycle then $G'$ has an undirected Hamilton cycle.
   Prove or Disprove: If $G'$ has an undirected Hamilton cycle then $G$ has a directed Hamilton cycle.

4. Prove that DIRECTED-HAM-CYCLE is NP-Complete. You may reduce from any of 3-CNF-SAT, CLIQUE, VERTEX-COVER, SUBSET-SUM, HAM-CYCLE, TSP.
5. The set intersection problem (SIP) is defined as follows: Given finite sets $A_1, A_2, \ldots, A_r$ and $B_1, B_2, \ldots, B_s$, is there a set $T$ such that

$$|T \cap A_i| \geq 1 \text{ for } i = 1, 2, \ldots, r$$

and

$$|T \cap B_j| \leq 1 \text{ for } j = 1, 2, \ldots, s?$$

Show that the set intersection problem is NP-complete. (Hint: Reduce from 3-CNF-SAT).

6. You are to prove that the 2-MIN-CLUSTER problem (defined below) is NP-Complete. You must use one of the following NP-complete problems for your reduction: CIRCUIT-SAT, SAT, 3-CNF-SAT, CLIQUE, VERTEX-COVER, SUBSET-SUM, SET-PARTITION, HAM-CYCLE, TSP.

You are given as input an undirected weighted graph $G = (V, E)$ and an integer $k$. For a partition of the vertices $V_1, V_2$ (so $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = V$), let $E' \subseteq E$ be the edges in which either both endpoints are in $V_1$ or both endpoints are in $V_2$. Then the weight of the partition, $w(V_1, V_2) = \sum_{e \in E'}$ weight of $e$. The question is whether or not there is a partition of the vertices $V_1, V_2$ such that $w(V_1, V_2) = k$.

7. For the following problem either prove the decision version is NP-complete or give a polynomial time algorithm. If it is in P give the most efficient algorithm you can for it.

You are given a tree $T$ and must compute a minimum sized vertex cover for $T$. 