CS 441T/539T Advanced Algorithms **Fall Semester, 2002**

More NP-completeness Practice Problems

November 7, 2002

1. Answer each of the following questions and briefly explain your answer. You are given as facts that 3-CNF-SAT and TSP (traveling salesman problem) are NP-complete and that there is a polynomial time algorithm for 2-CNF-SAT.

   (a) True or false: 3-CNF-SAT $\leq_p$ TSP.

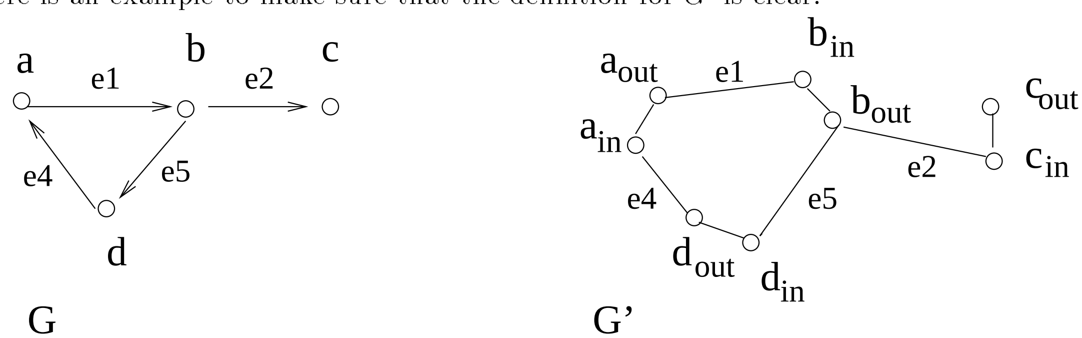
   (b) True or false: If P $\neq$ NP, then no NP-complete problem can be solved in polynomial time.

   (c) True or false: 3-CNF-SAT $\leq_p$ 2-CNF-SAT (assume that P $\neq$ NP).

2. A *feedback vertex set* in a directed graph $G = (V, E)$ is a subset V' of V such that V' contains at least one vertex from each directed cycle in G. The *feedback vertex set problem* (FVS) is: Given a directed graph G and an integer ℓ, does G have a feedback vertex set with at most ℓ vertices? Prove that VERTEX-COVER $\leq_p$ FVS. Then answer the below question.

   Does it follow from the fact that VERTEX-COVER $\leq_p$ FVS that FVS is NP-complete? Why?

3. The Hamilton cycle problem (HAM-CYCLE) that we have studied is defined for undirected graphs. We now consider the directed Hamilton cycle problem (DIRECTED-HAM-CYCLE) in which you are given a directed graph G and asked if there is a directed cycle that visits every vertex exactly once. In this problem we consider a proposed reduction to show DIRECTED-HAM-CYCLE $\leq_p$ HAM-CYCLE. Here is the proposed input transformation function f. Let $G = (V, E)$ be the directed graph that is input for DIRECTED-HAM-CYCLE. We define the undirected graph $G' = (V', E')$ for HAM-CYCLE as follows. For each vertex $v \in V$ there are two vertices v_{in} and v_{out} placed in V'. E' contains $|V| + |E|$ edges. First, for each $v \in V$ an edge is placed between v_{in} and v_{out}. Then for a directed edge from u to v in E, we place an undirected edge from u_{out} to v_{in} in E'. Here is an example to make sure that the definition for G' is clear.



   Prove or Disprove: If G has a directed Hamilton cycle then G' has an undirected Hamilton cycle.

   Prove or Disprove: If G' has an undirected Hamilton cycle then G has a directed Hamilton cycle.

4. Prove that DIRECTED-HAM-CYCLE is NP-Complete. You may reduce from any of 3-CNF-SAT, CLIQUE, VERTEX-COVER, SUBSET-SUM, HAM-CYCLE, TSP.

5. The *set intersection* problem (SIP) is defined as follows: Given finite sets $A_1, A_2, \ldots, A_r$ and $B_1, B_2, \ldots, B_s$, is there a set T such that

$$|T \cap A_i| \geq 1 \text{ for } i = 1, 2, \ldots, r$$

and

$$|T \cap B_j| \leq 1 \text{ for } j = 1, 2, \ldots, s?$$

Show that the set intersection problem is NP-complete. (Hint: Reduce from 3-CNF-SAT).

6. You are to prove that the 2-MIN-CLUSTER problem (defined below) is NP-Complete. You must use one of the following NP-complete problems for your reduction: CIRCUIT-SAT, SAT, 3-CNF-SAT, CLIQUE, VERTEX-COVER, SUBSET-SUM, SET-PARTITION, HAM-CYCLE, TSP.

   You are given as input an undirected weighted graph $G = (V, E)$ and an integer k. For a partition of the vertices V_1, V_2 (so $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = V$), let $E' \subset E$ be the edges in which either both endpoints are in V_1 or both endpoints are in V_2. Then the weight of the partition, $w(V_1, V_2) = \sum_{e \in E'}$ weight of e. The question is whether or not there is a partition of the vertices V_1, V_2 such that $w(V_1, V_2) = k$.

7. For the following problem either prove the decision version is NP-complete or give a polynomial time algorithm. If it is in P give the most efficient algorithm you can for it.

   You are given a tree T and must compute a minimum sized vertex cover for T.