

Exam 2

November 14, 2002

IF YOU HAVE QUESTIONS ABOUT THE STATEMENT OF ANY PROBLEM, PLEASE COME TO THE FRONT OF THE ROOM AND ASK.

IF YOU ARE STUCK ON ANY PROBLEM CONSIDER GETTING A HINT.

1. (10 pts) Give a linear program that could be used to find an optimal solution to the following problem. You do NOT need to actually solve the linear program.

The ACME Mine Company owns two different mines that produce an ore which, after being crushed, is graded into three classes: high, medium and low. The company has contracted to provide a smelting plant with 12 tons of high-grade ore, 8 tons of medium-grade ore, and 24 tons of low-grade ore per week. The two mines have different operating characteristics as detailed below:

Mine	Cost per day (\$1000)	Production (tons/day)		
		High	Medium	Low
X	180	6	3	4
Y	160	1	1	6

The goal is to determine how many days per week each mine should be operated to minimize costs will still fulfilling the smelting plant contract.

2. (10 pts) For each of the following questions you should indicate which of the following 4 possible answers holds: “known to be true”, “known to be false”, “True if and only if $P=NP$ ”, “True if and only if $P \neq NP$ ”.

(a) If $A \leq_p B$ and B is in P , then A is in P .

(b) If $A \leq_p B$ and B is in NP , then A is in NP .

(c) Every problem in NP is NP -complete.

(d) Every NP -complete problem is in NP .

(e) There are problems in NP that are not NP -complete.

(f) No NP -complete problem can be solved in polynomial time.

(g) No problem in NP can be solved in polynomial time.

(h) Integer Programming \leq_p Linear Programming.

(i) Linear Programming \leq_p Integer Programming.

(j) If $A \leq_p B$, A is in NP , and B is NP -complete, then A is NP -complete.

3. (15 pts) Consider the following Restaurant Critic problem. There are n restaurants you need to compare. You have asked for many opinions with each one modeled by a weighted directed edge. The edge e from i to j with weight $w(e)$ indicates that j is better than i and the weight $w(e)$ indicates how strongly this belief is held.

Thus the opinions are modeled by a weighted directed graph $G = (V, E)$. Your goal is to produce a directed acyclic graph (i.e. a directed graph without any cycles) $G' = (V, E')$ where $E' \subseteq E$ such that $\sum_{e \in E'} w(e)$ is maximized. In other words you want to provide a non-conflicting set of recommendations with as much weight as possible. Before continuing be sure you understand this question. Please come to the front if you do not.

Consider the following approximation algorithm for this problem.

- (a) Assign an arbitrary ordering to the vertices in V .
- (b) Let $E_1 = \{(i, j) \mid i < j\}$ and let $E_2 = \{(i, j) \mid i > j\}$. That is if you put the vertices in order from left to right, E_1 are the edges that go to the right and E_2 are the edges that go to the left.
- (c) If $\sum_{e \in E_1} w(e) \geq \sum_{e \in E_2} w(e)$ then return E_1 . Else return E_2 .

Prove the above is a 2-approximation.

4. (20 points) The restaurant critic problem given in the last problem is an optimization problem. Below define RC as a decision version of this problem.

Now prove that RC is NP-complete. To do this you should make use of the fact that FVS (defined below) is NP-complete. A *feedback vertex set* in directed graph $G = (V, E)$ is a subset $V' \subseteq V$ such that V' contains at least one vertex from each directed cycle in G . The *feedback vertex set problem* (FVS) is: Given a directed graph G and an integer ℓ , does G have a feedback vertex set with at most ℓ vertices?

Additional Work for Problem 4

THE CS 441T EXAM ENDS HERE. (If a CS 441T student does the next problem we will note it as extra credit, but no points will be added to your total. Hence CS 441T students should do all they can on problems 1-4 before even considering problem 5.)

THIS PROBLEM IS REQUIRED ONLY FOR CS 539 STUDENTS.

For this problem you can only use the following problems for your reduction: SAT, 3-CNF-SAT, CLIQUE, VERTEX-COVER, SUBSET-SUM, PARTITION, HAM-CYCLE, TSP.

5. (15 points) Consider a variation of the maximum flow problem where the flow on each edge must be either 0 or the capacity of the edge. More specifically, in the MAX-SATURATED-FLOW problem you are given a directed weighted graph $G = (V, E)$ with a specified source vertex $s \in V$ and a specified sink vertex $t \in V$. You are also given a desired flow value f . The question is whether or not you can assign each edge e a flow value of either 0 or $w(e)$ such that the flow into t is at least f . As in the standard maximum flow problem, for each $v \in V - \{s, t\}$ the flow into v must equal the flow out of v .

Prove that MAX-SATURATED-FLOW is NP-hard.

Problem	Points Possible	Points Received
1	10	
2	10	
3	20	
4	15	
5*	15	
total	55 + 10	