

More Practice Problems

December 5, 2001

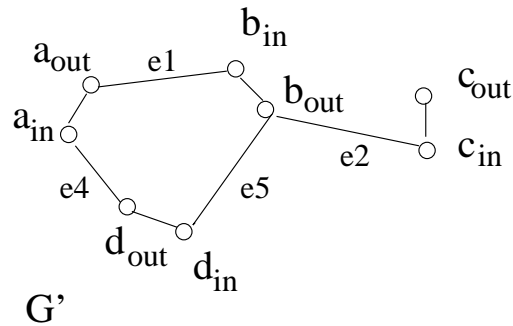
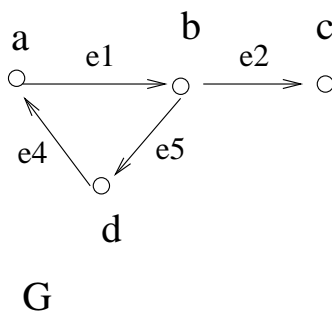
1. Answer each of the following questions and briefly explain your answer. You are given as facts that 3-CNF-SAT and TSP (traveling salesman problem) is NP-complete and that there is a polynomial time algorithm for 2-CNF-SAT.

- (a) True or false: $3\text{-CNF-SAT} \leq_p \text{TSP}$.
 (b) True or false: If $P \neq \text{NP}$, then no problem in NP can be solved in polynomial time.
 (c) True or false: $3\text{-CNF-SAT} \leq_p 2\text{-CNF-SAT}$ (assume that $P \neq \text{NP}$).

2. A *feedback vertex set* in a directed graph $G = (V, E)$ is a subset V' of V such that V' contains at least one vertex from each directed cycle in G . The *feedback vertex set problem* (FVS) is: Given a directed graph G and an integer ℓ , does G have a feedback vertex set with at most ℓ vertices? Prove that $\text{VERTEX-COVER} \leq_p \text{FVS}$. Then answer the below question.

Does it follow from the fact that $\text{VERTEX-COVER} \leq_p \text{FVS}$ that FVS is NP-complete? Why?

3. The Hamilton cycle problem (HAM-CYCLE) that we have studied is defined for undirected graphs. We now consider the directed Hamilton cycle problem (DIRECTED-HAM-CYCLE) in which you are given a directed graph G and asked if there is a directed cycle that visits every vertex exactly once. In this problem we consider a proposed reduction to show $\text{DIRECTED-HAM-CYCLE} \leq_p \text{HAM-CYCLE}$. Here is the proposed input transformation function f . Let $G = (V, E)$ be the directed graph that is input for DIRECTED-HAM-CYCLE. We define the undirected graph $G' = (V', E')$ for HAM-CYCLE as follows. For each vertex $v \in V$ there are two vertices v_{in} and v_{out} placed in V' . E' contains $|V| + |E|$ edges. First, for each $v \in V$ an edge is placed between v_{in} and v_{out} . Then for a directed edge from u to v in E , we place an undirected edge from u_{out} to v_{in} in E' . Here is an example to make sure that the definition for G' is clear.



Prove or Disprove: If G has a directed Hamilton cycle then G' has an undirected Hamilton cycle.

Prove or Disprove: If G' has an undirected Hamilton cycle then G has a directed Hamilton cycle.

Based on your above answers (take a guess above if needed so you can answer this part), does this reduction prove that $\text{DIRECTED-HAM-CYCLE} \leq_p \text{HAM-CYCLE}$?

4. Prove that DIRECTED-HAM-CYCLE is NP-Complete. You may reduce from any of 3-CNF-SAT, CLIQUE, VERTEX-COVER, SUBSET-SUM, HAM-CYCLE, TSP.
5. The *set intersection* problem (SIP) is defined as follows: Given finite sets A_1, A_2, \dots, A_r and B_1, B_2, \dots, B_s , is there a set T such that

$$|T \cap A_i| \geq 1 \text{ for } i = 1, 2, \dots, r$$

and

$$|T \cap B_j| \leq 1 \text{ for } j = 1, 2, \dots, s?$$

Show that the set intersection problem is NP-complete. (Hint: Reduce from 3-CNF-SAT).

6. You are to prove that the 2-MIN-CLUSTER problem (defined below) is NP-Complete. You must use one of the following NP-complete problems for your reduction: CIRCUIT-SAT, SAT, 3-CNF-SAT, CLIQUE, VERTEX-COVER, SUBSET-SUM, SET-PARTITION, HAM-CYCLE, TSP.

You are given as input an undirected weighted graph $G = (V, E)$ and an integer k . For a partition of the vertices V_1, V_2 (so $V_1 \cap V_2 = \emptyset$ and $V_1 \cup V_2 = V$), let $E' \subset E$ be the edges in which either both endpoints are in V_1 or both endpoints are in V_2 . Then the weight of the partition, $w(V_1, V_2) = \sum_{e \in E'} \text{weight of } e$. The question is whether or not there is a partition of the vertices V_1, V_2 such that $w(V_1, V_2) = k$.

7. For the following problem either prove the decision version is NP-complete or give a polynomial time algorithm. If it is in P give the most efficient algorithm you can for it.

You are given a tree T and must compute a minimum sized vertex cover for T .

8. Consider the problem of merging a sorted list X of n items with a sorted list Y of 2 items. Give the best lower bound you can for the number of comparisons required in the worst-case by any comparison-based algorithm to solve this problem.
9. Give a lower bound on the number of cuts necessary to slice a block of cheese into n pieces where a piece can be any shape. A slice is defined as a plane passing through the cheese.