CS 201 Formal Foundations of Computer Science	**Fall Semester, 2000**

Homework Assignment 3

September 20, 2000 — *Due Date: Sept. 27 (Quiz on Sept. 25)*

Practice Exercises

1. For the following two arguments, prove whether or not each is valid.

   a.
   (1) $p \wedge q$
   (2) $\neg r \rightarrow \neg p$
   (3) $(\neg r \vee s) \vee t$
   (4) $\neg s$
   $\therefore\ t$

   b.
   (1) $p \rightarrow r$
   (2) $p \vee s$
   (3) $q \rightarrow r$
   (4) $\neg s$
   $\therefore\ q$

2. Prove that the following is a valid argument.

   $$\begin{array}{c} p \oplus q \\ \neg p \\ \hline \therefore\ q \end{array}$$

   That is, you must prove that $((p \oplus q) \wedge \neg p) \rightarrow q$ is a tautology. You may use any proof technique, including a truth table, for this problem. Remember, that for any new rule of logic you want to introduce (throughout this course), you **must** first prove it is valid.

3. Write the following argument in symbolic form, and then prove whether or not is a valid argument: "If today is Friday, then I have a test in Computer Science or a test in Economics. If my Economics professor is sick, then I will not have a test in Economics. Today is Friday and my Economics professor is sick. Therefore, I have a test in Computer Science."

   Let f be "Today is Friday," c be "I have a test in Computer Science," e be "I have a test in Economics," and s be "My economics professor is sick."

4. Show that the following argument is valid:
   (1) $\forall x\ P(x) \rightarrow Q(x)$
   (2) $\exists x\ \neg Q(x)$
   (3) $\forall x\ P(x) \vee R(x)$
   (4) $\forall x\ R(x) \rightarrow S(x)$
   $\therefore\ \exists x\ S(x)$

   Would this argument still be valid if premise (3) was changed to $\exists x\ P(x) \vee R(x)$? Prove that your answer is right.

5. Let $P(x)$ be "x is a clear explanation," let $Q(x)$ be "x is satisfactory," and let $R(x)$ be "x is an excuse." Express each of the following statements using quantifiers, logical connectives, and $P(x), Q(x)$, and $R(x)$.

   (a) All clear explanations are satisfactory.
   (b) Some excuses are unsatisfactory.
   (c) Some excuses are not clear explanations.
   (d) Prove or disprove: (c) follows from (a) and (b).

6. Prove that if n is a positive integer such that the sum of its divisors is $n+1$, then n is prime. (*Hint: Use an indirect proof. Also, note that 1 is not a prime number. You may want a special case for it.*)

Problems to Submit

1. (8 pts) Write the following argument in symbolic form, and then prove whether or not is a valid argument: "If there is a chance of rain or her red headband is lost, then Lois will not mow her lawn. Whenever the temperature is over 80 degrees, there is no chance for rain. Today the temperature is 85 degrees and Lois is wearing her red headband. Therefore, Lois will mow her lawn."

   Let r be "There is a chance of rain," l be "Lois' red headband is lost," m be "Lois will mow her lawn," and h be "the temperature is over 80 degrees."

2. (14 pts) For the following two arguments, prove whether or not each is valid.

   a.
   (1) $p \rightarrow s$
   (2) $(\neg y \wedge \neg w) \vee \neg s$
   (3) $p \wedge q$
   (4) $\neg z \rightarrow y$

   $\therefore\ q \wedge z$

   b.
   (1) $(\neg p \wedge \neg s) \rightarrow q$
   (2) $(\neg p \wedge \neg w) \rightarrow \neg r$
   (3) $y \rightarrow \neg q$
   (4) $y \vee r$

   $\therefore\ p \vee s \vee w$

   Hint: You might find a proof by contradiction to be useful.

3. (14 pts) Prove whether or not each of (a) and (b) is a tautology. Then answer (c). *(Hint: You may find an indirect proof to be useful.)*

   (a) $[\ (\forall x\ P(x)) \rightarrow (\exists y\ Q(y))\ ] \ \rightarrow\ [\ \forall x \exists y\ (P(x) \rightarrow Q(y))\ ]$

   (b) $[\ \forall x \exists y\ (P(x) \rightarrow Q(y))\ ] \ \rightarrow\ [\ (\forall x\ P(x)) \rightarrow (\exists y\ Q(y))\ ]$

   (c) Are $(\forall x\ P(x)) \rightarrow (\exists y\ Q(y))$ and $\forall x \exists y\ (P(x) \rightarrow Q(y))$ logically equivalent? Explain.

4. (14 pts) Let $D(x)$ be "x is a duck," let $W(x)$ be "x is willing to waltz,", let $O(x)$ be "x is an officer", and let $P(x)$ be "x is one of my poultry." Express each of the following statements using quantifiers, logical connectives, and $P(x)$, $Q(x)$, and $R(x)$.

   (a) No ducks are willing to waltz.

   (b) No officers ever decline to waltz.

   (c) Some of my poultry are ducks.

   (d) Not all of my poultry are officers.

   (e) Prove or disprove: (d) follows from (a), (b), and (c).

Challenge Problem

Suppose we have three people, Alice, Bob, and you. Alice and Bob have both taken CS 201 and received an "A+", so their logical reasoning is both flawless and instantaneous. I have three red hats and two white hats. I'll put one hat on each person (Alice, Bob, and you). You can see neither your own hat nor the unused hats. However, each of you can see the hats worn by the other two people. First I ask Alice, "Alice, what color is your hat?" She replies, "I don't know." Next I ask Bob, "Now, Bob, what color is your hat?" He replies, "I don't know." At this point can you tell me the color of your hat? If so, what color is it?

Convince me that your answer is correct.