

Homework Assignment 2

September 13, 2000

Due Date: Sept. 20 (Quiz on Sept. 18)

As a reminder you need not turn in the practice exercises. However, to provide *strong* encouragement to do them, on September 18th, one of the below practice exercises (selected by the roll of a die) will be given as a closed-book quiz.

Homeworks should be neat and should *not* be done in red ink. Also don't forget to put your name. If you are unable to attend class then turn in your homework assignment BEFORE (i.e. by 2:15pm) class in my box (labeled S. Goldman) in Bryan 509C.

Practice Exercises

- Let
 - $B(x)$ be the predicate “ x is a bee”,
 - $F(x)$ be the predicate “ x is a flower”,
 - $L(x, y)$ be the predicate “ x likes y ”
 where the universe of discourse for x and y are all objects in the world. Express each of the below statements using these predicates, quantifiers, and logical connectives.
 - Some bee dislikes all flowers.
 - No bee likes only flowers.
- Consider the following predicates: $S(x)$ is “ x is a student,” $C(x)$ is “ x is curious,” $A(x)$ is “ x is adventuresome,” and $Q(x, y)$ is “ x asks questions of y .” Express each of the following statements using quantifiers, logical connectives, and the above predicates where the universe of discourse for both x and y are all people.
 - There are some students who do not ask questions.
 - All curious people asks questions, and some curious people are also adventuresome.
- Explain why the negation of “Some student in my class uses e-mail” is not “Some student in my class does not use e-mail”. Then write (in English) the clearest statement you can that expresses the negation of “Some student in my class uses e-mail.”
- Use the laws of logic to prove the following two expressions are logically equivalent.

$$\exists x \forall y [P(x) \wedge ((y \neq x) \rightarrow (\neg P(y)))] \quad \text{and} \quad \exists x [P(x) \wedge \neg \exists y (P(y) \wedge (y \neq x))]$$

Then summarize in English what this statement says.

- Prove whether or not the following two statements are true.
 - $(\forall x P(x)) \vee (\forall x Q(x)) \iff \forall x (P(x) \vee Q(x))$
 - $(\forall x P(x)) \vee (\forall x Q(x)) \iff \forall x \forall y (P(x) \vee Q(y))$
- Prove whether or not the following is a tautology.

$$[(\exists x P(x)) \rightarrow (\forall x Q(x))] \rightarrow [\forall x (P(x) \rightarrow Q(x))]$$

Problems to Submit

1. (9 pts) Express each of the below statements using the provided predicates, quantifiers, and logical connectives. Let

$C(x)$ be the predicate “ x is a Corvette”,
 $F(x)$ be the predicate “ x is a Ferrari”,
 $P(x)$ be the predicate “ x is a Porsche”,
 $S(x, y)$ be the predicate “ x is slower than y ”

- (a) Nothing is both a Corvette and a Ferrari.
(b) Some Porsche is slower than only Ferraris.
(c) All Ferraris are slower than some Corvette.

2. (12 pts) Let

$L(x)$ be the predicate “ x is a lawyer”,
 $J(x)$ be the predicate “ x is a judge”,
 $A(x, y)$ be the predicate “ x admires y ”

where the universe of discourse for x and y are all people. Express each of the below statements using these predicates, quantifiers, and logical connectives.

- (a) Some lawyer admires only judges.
(b) All judges admire only judges.
(c) Only judges admire judges.
(d) Nobody admires all lawyers.

3. (10 pts) Use the laws of logic to prove the following two expressions are logically equivalent.

$$\exists u \exists b (L(b, u) \wedge \forall r (I(r, b) \rightarrow W(r))) \iff \exists u \neg \forall b \exists r (\neg I(r, b) \rightarrow \neg L(b, u)) \wedge \neg(L(b, u) \wedge W(r))$$

4. (14 pts) For each of the following prove whether or not it is a tautology.

- (a) $[(\forall x P(x)) \vee (\forall x Q(x))] \rightarrow \forall x (P(x) \vee Q(x))$
(b) $[\forall x (P(x) \vee Q(x))] \rightarrow [(\forall x P(x)) \vee (\exists y Q(y))]$
(c) $[\forall x (P(x) \rightarrow Q(x))] \rightarrow [(\exists x P(x)) \rightarrow (\forall x Q(x))]$.

Challenge Problem: Let $P(x)$ denote that x is a politician, and $Q(x, y)$ denote that x quotes y where the universe of discourse for x and y are all people. Express each of the below statements using these predicates, quantifiers, and logical connectives.

1. Every politician quotes someone, but no politician is quoted by every one of the politicians that she quotes.
2. If a politician quotes any politician that does not quote him, he quotes every politician that quotes no politician.