CS 201 Formal Foundations of Computer Science **Fall Semester, 2000**

Homework Assignment 1

September 6, 2000 *Due Date: Sept. 13 (Quiz on Sept. 11)*

As a reminder you need not turn in the practice exercises. However, to provide *strong* encouragement to do them, on September 9th, one of the below practice exercises (selected by the roll of a die) will be given as a closed-book quiz. Please review the collaboration policy (in the "Course Information" Handout or on the web page). If you have any questions, contact me.

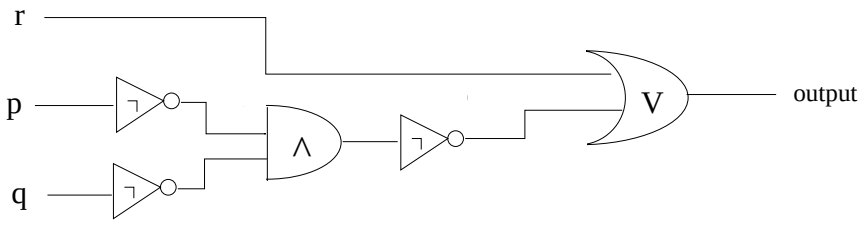
Homeworks should be neat and should *not* be done in red ink. Also don't forget to put your name. If you are unable to attend class then turn in your homework assignment BEFORE (i.e. by 2:15pm) class in my box (labeled S. Goldman) in Bryan 509C.

Practice Exercises

1. Which of the following are propositions:
   (a) The Cardinals are going to win the 2000 World Series.
   (b) $x/3$ is an integer.
   (c) Do not pass go.
   (d) St. Louis is hot in the summer.
   (e) Is there is life on mars?
2. Let p be the proposition "You forgot to set your alarm." Let q be the proposition "You miss the final exam." And let r be the proposition "You pass the course." Formulate in English the following compound propositions.
   (a) $r \rightarrow \neg q$
   (b) $(p \rightarrow q) \wedge (q \rightarrow \neg r)$
   (c) $(p \wedge q) \vee (\neg q \wedge r)$
3. Let s be the proposition "you get a speeding ticket" and let f be the proposition "you drive over 65 mph." Express each of the following statements using s, f, and logical connectives.
   (a) You do not drive over 65 mph or you get a speeding ticket, but not both.
   (b) You have driven over 65 mph if you get a speeding ticket.
   (c) Driving over 65 mph will get you a speeding ticket.
   (d) You get a speeding ticket, but you do not drive over 65 mph.
   (e) Whenever you get a speeding ticket, you are driving over 65 mph.
4. Which of the following compound propositions are a tautology? You may use a truth table, but are not required to.
   To prove a proposition is NOT a tautology you need just give one setting for p and q for which the compound proposition is false. You may want to use any tautology below as practice in proving that a proposition is a tautology WITHOUT using a truth table.
   (a) $(p \rightarrow q) \rightarrow (q \rightarrow p)$
   (b) $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$
   (c) $q \rightarrow (\neg p \vee \neg q)$

5. For the compound proposition, $\neg p \wedge \neg q \wedge (\neg r \rightarrow p)$, find an equivalent expression which uses only $\wedge$ and $\neg$ and which is as simple as possible.

6. Consider the following circuit that uses negation gates, AND gates and OR gates.



Notice that the logical expression describing the functionality of this circuit is given by: $r \vee \neg(\neg p \wedge \neg q)$. The given circuit uses 5 gates. Use the rules of logic to simplify the design so that only 2 gates are required.

You should give the logical expression and the circuit diagram (like above) for your new design that uses only 2 gates.

Problems to Submit

1. (10 pts) Let

   g be the proposition "your have a green parking permit",
   b be the proposition "your have a blue parking permit",
   x be the proposition "you can legally park in the XYZ garage",
   f be the proposition "you paid the entrance fee".

   Express each of the below statements using these propositions and logical connectives.

   (a) To park in the XYZ garage you must pay the entrance fee.

   (b) You can legally park in the XYZ garage if you paid an entrance fee or you have a blue parking permit.

   (c) Having a green parking permit is necessary to legally park in the XYZ garage.

   (d) To pay an entrance free it is sufficient to have a blue permit.

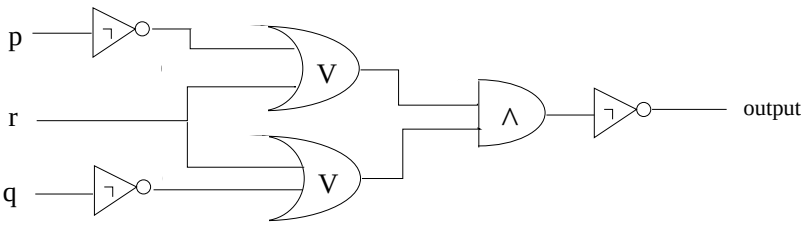
   (e) Having a blue or green parking permit is necessary to legally park in the XYZ garage.

2. (10 pts) Prove that the following are tautologies <u>without</u> using a truth table.

   (a) $((p \rightarrow q) \wedge (\neg p \rightarrow q)) \rightarrow q$

   (b) $((\neg p \wedge r) \rightarrow ((q \vee \neg r) \wedge r)) \vee r$

3. (10 pts) Consider the following circuit that uses negation gates, AND gates and OR gates.



(a) Write a logical expression describing the functionality of this digital circuit.

(b) Notice that the given circuit using 6 gates. Use the laws of logic to simplify the design so that only 3 gates are required to obtain the same logical function. Show your work! Also you are *required* to the logical expression and the circuit diagram (like above) for your new design that uses only 3 gates.

4. (10 pts) Prove that $(p_1 \vee p_2 \vee p_3) \rightarrow q$ and $(p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge (p_3 \rightarrow q)$ are logically equivalent (without using a truth table).

Extra Credit Problem:

A man who was captured by savages was promised his freedom if he could determine with a single "yes or no" question the color of the tribe's idol. He knew the idol was either white or black. Unfortunately, the tribe contained two kinds of individuals: liars, who invariably gave the wrong answer to any question they were asked, and truth-tellers who invariably gave the right answer.

Provide a single "yes or no" question that will enable him to determine the color correctly and prove to him that your proposed question will serve the desired role.