Optical flow estimation in omnidirectional images using wavelet approach

Cédric Demonceaux[1,2] and Djemâa Kachi-Akkouche[1]

[1]C.R.E.A., EA 3299
7, rue du moulin neuf
80000 Amiens, France

[2]L.A.M.F.A., UMR 6140
33, rue Saint-Leu
80039 Amiens Cedex 1, France

cedric.demonceaux@u-picardie.fr

Abstract

The motion estimation computation in the image sequences is a significant problem in image processing. Many researches were carried out on this subject in the image sequences with a traditional camera. These techniques were applied in omnidirectional image sequences. But the majority of these methods are not adapted to this kind of sequences. Indeed they suppose the flow is locally constant but the omnidirectional sensor generates distortions which contradict this assumption. In this paper, we propose a fast method to compute the optical flow in omnidirectional image sequences. This method is based on a Brightness Change Constraint Equation decomposition on a wavelet basis. To take account of the distortions created by the sensor, we replace the assumption of flow locally constant used in traditional images by a hypothesis more appropriate.

1 Introduction

This paper studies the problem of the optical flow estimation in the omnidirectional image sequences. Our omnidirectional images are obtained with a parabolic mirror located above a traditional camera. This type of sensor provides an omnidirectional (360 degree) field of view of the scene observed. They are widely used in many robotics applications: localization, 3D reconstruction, obstacle detection...[12]. The computation of the optical flow will enable us to determine ego-motion information as well as a general motion information of the scene [5].

In traditional planar images, many researches deal with optical flow estimation. These methods are classified in three classes: matching techniques, energy-based methods, differential techniques [1]. These methods were applied in omnidirectional images. Stratmann [10] proved in this case the estimation optical flow estimation with the method proposed by Fleet [4] is quietly robust and precise. Because of its computer complexity, it is not possible to use such method in robotics applications. The author concluded the method proposed by Lucas and Kanade [8] makes a good compromise between rapidity and the estimation accuracy. However, this method is not completely adapted to the omnidirectional image sequences. Indeed this technique supposes the flow is constant on an area of the image what is never checked in the omnidirectional images since the sensor generates distortions of the scene observed. To solve this problem, Daniilidis [3] proposed to take account of the sensor geometry by computing the optical flow starting from spherical co-ordinate. It enables him to obtain an equation of the optical flow constraint different from that used usually in the traditional image sequences.

In this article, we develop a method which approaches the method of Lucas and Kanade [8] by taking into account the distortions created by the sensor. To fulfill the requirement of real times application, we follow the method proposed by Bernard [2] which carries out a good compromise between precision of optical flow estimation and speed of computation. We will show this method requires a multiresolution approach to compute the optical flow. Then we will describe the functions used to estimate the flow. Finally we will show our hypothesis gives better results for the omnidirectional images than optical flow assuming that is locally constant.

2 Optical flow computation

The central assumption in optical flow computation is the intensity is constant along the times for each physic point in the image. So if we assume this grey-value preservation, we can easily derive and obtain the well known Brightness Change Constraint Equation :

$$\nabla I((x,y),t).v((x,y),t) + \frac{\partial I((x,y),t)}{\partial t} = 0 \qquad (1)$$

$\nabla I((x,y),t)$ and $\frac{\partial I((x,y),t)}{\partial t}$ are respectively gradient and temporal derivative of the intensity function $I((x,y),t)$ and $v((x,y),t)$ is the field of optical flow.

Considering this equation (1), we can only estimate the normal velocity component denoted by :

$$\overline{v}_n = -\frac{1}{\|\nabla I\|}.\frac{\partial I}{\partial t} \qquad (2)$$

To overcome this problem called "aperture problem", Horn and Schunck [6] introduce the smoothness constraints. Tretiak and Pastor [11] assumed the image gradient conservation in order to get two linear equations. The method described in our work uses the wavelet approach to obtain more equations to estimate the optical flow (v_1, v_2).
Let us consider the wavelets basis $(\Psi^n)_{i=1\cdots N}$ in $L^2(\mathbf{R}^2)$ centered around the origin $(0,0)$, and let us consider the N functions centered around the point $u = (u_1, u_2)$ defined as

$$\Psi_u^n(x,y) = \Psi^n(x - u_1, y - u_2). \qquad (3)$$

Taking the inner product of (1) with Ψ_u^n, we obtain the following system :

$$\langle \nabla I.v + \frac{\partial I}{\partial t}, \Psi_u^n \rangle = 0 \qquad \forall n = 1 \cdots N, \qquad (4)$$

where

$$\langle f, g \rangle = \int\int f(x)\overline{g(x)}dxdy. \qquad (5)$$

And finally, that leads to

$$\langle \frac{\partial I}{\partial x}v_1, \Psi_u^n \rangle + \langle \frac{\partial I}{\partial y}v_2, \Psi_u^n \rangle + \langle \frac{\partial I}{\partial t}, \Psi_u^n \rangle = 0, \qquad \forall n = 1..N. \qquad (6)$$

To perform the computations, we have to make one more assumption on the unknowns (v_1, v_2).
When we deal with planar images, it is usual to assume that the flow is locally constant [2], [8]. With the omnidirectional images, this hypothesis is never valid. Instead of the constancy assumption of the velocity, we propose to modelize the optical flow $v(x,y) = (v_1(x,y), v_2(x,y))$ on the support of the wavelet Ψ_u^n with an affine model defined by :

$$\begin{array}{c} v_1(x,y) = ax + by + c \\ v_2(x,y) = dx + ey + f \\ \forall (x,y) \in supp\Psi_u^n \qquad \forall n = 1..N. \end{array} \qquad (7)$$

With this model, we suppose the observed motion is not locally constant but it features rotation, translation and dilatation or any combination of these basic motions.
By substituting the affine model in the system (6) we obtain the system :

$$\begin{array}{l} a\langle x\frac{\partial I}{\partial x}, \Psi_u^n \rangle + b\langle y\frac{\partial I}{\partial x}, \Psi_u^n \rangle + c\langle \frac{\partial I}{\partial x}, \Psi_u^n \rangle + \\ d\langle x\frac{\partial I}{\partial y}, \Psi_u^n \rangle + e\langle y\frac{\partial I}{\partial y}, \Psi_u^n \rangle + f\langle \frac{\partial I}{\partial y}, \Psi_u^n \rangle + \\ \langle \frac{\partial I}{\partial t}, \Psi_{jk}^n \rangle = 0 \qquad \forall n = 1..N. \end{array} \qquad (8)$$

Integrating by parts, we obtain a new system where the coefficient (a,b,c,d,e,f) are the unknowns, this system reads as follows:

$$\begin{array}{l} a\left(\langle xI, \frac{\partial \Psi_u^n}{\partial x} \rangle + \langle I, \Psi_u^n \rangle\right) + b\langle yI, \frac{\partial \Psi_u^n}{\partial x} \rangle + c\langle I, \frac{\partial \Psi_u^n}{\partial x} \rangle + \\ d\langle xI, \frac{\partial \Psi_u^n}{\partial y} \rangle + e\left(\langle yI, \frac{\partial \Psi_u^n}{\partial y} \rangle + \langle I, \Psi_u^n \rangle\right) + f\langle I, \frac{\partial \Psi_u^n}{\partial y} \rangle = \\ \langle \frac{\partial I}{\partial t}, \Psi_u^n \rangle \qquad \forall n = 1..N. \end{array} \qquad (9)$$

So the "aperture problem" can be solved if N ≥ 6. The optical flow estimation at the point $u = (u_1, u_2)$ is equivalent to estimate the 6 unknowns (a,b,c,d,e,f). In the following we rewrite our system (9) in this as follows

$$M_u V = X_u \qquad \text{with} \qquad V = (a,b,c,d,e,f)^T. \qquad (10)$$

3 Temporal aliasing

To relax the hypothesis (7), we would like to take wavelets with the smallest possible support. However the approximation of the temporal derivative $\frac{\partial I}{\partial t}$ with the finite difference $I(t+1) - I(t)$ forces to assume the converse hypothesis. Indeed if we suppose the image sequence is a uniform translation of vector v.

$$I((x,y),t) = I(x - tv, y - tv). \qquad (11)$$

Let us consider the functions ψ^n on various scales

$$\Psi_{us}^n(x,y) = s^{-1}\Psi^n(\frac{x-u_1}{s}, \frac{y-u_2}{s}) \qquad (12)$$

and the approximation

$$\frac{\partial I}{\partial t} \simeq I(t+1) - I(t) \qquad (13)$$

[2] show that if we replace $\frac{\partial I}{\partial t}$ by $I(t+1) - I(t)$ in (9), this equation is again valid if vs^{-1} is not large, i.e there exists M such that

$$\|v\| < Ms \qquad (14)$$

Consequently it is necessary to make a balance between the hypothesis (7) and (14). We do not know the flow we want to compute, then we must deal with wavelets which support varies. This can be performed with a suitable version of the well-known multiresolution method. In coarse grid we compute large displacements and we refine progressively to compute the low speeds.

4 Multiresolution

To apply multiresolution method we consider the discrete wavelet basis defined as follow:

$$\psi_{jk}^n(x,y) = 2^j\psi^n(2^j x - k_1, 2^j y - k_2), \qquad (15)$$

where $k = (k_1, k_2)$ and j is a resolution index. For each j fixed and for each k, we obtain N equations which allow to compute the flow at the points $(2^j k_1, 2^j k_2)$ by solving the system

$$
\begin{aligned}
&a\left(\langle xI, \frac{\partial \Psi^n_{jk}}{\partial x}\rangle + \langle I, \Psi^n_{jk}\rangle\right) + b\langle yI, \frac{\partial \Psi^n_{jk}}{\partial x}\rangle \\
&+c\langle I, \frac{\partial \Psi^n_{jk}}{\partial x}\rangle + d\langle xI, \frac{\partial \Psi^n_{jk}}{\partial y}\rangle + e\left(\langle yI, \frac{\partial \Psi^n_{jk}}{\partial y}\rangle + \langle I, \Psi^n_{jk}\rangle\right) \\
&+f\langle I, \frac{\partial \Psi^n_{jk}}{\partial y}\rangle = \langle \frac{\partial I}{\partial t}, \Psi^n_{jk}\rangle \qquad \forall n = 1..N.
\end{aligned} \tag{16}
$$

We denote this system as

$$M^j_k V^j_k = X^j_k \qquad \text{with} \qquad V^j_k = (a, b, c, d, e, f)^T. \tag{17}$$

For each k, we distinguish 3 possibilities: either the system is badly conditioned ($Cond(M^j_k V^j_k) > Cst$), or the solution of the system (17) does not fill the assumption (14), or the solution is valid. In both first cases, the flow cannot be calculated in this point and in this resolution. In the last case, we estimate that the solution of the system (17) is a measure of the optical flow at the point $(2^j k_1, 2^j k_2)$ as long as we cannot perform another computation at this point in a finer resolution (where (7) is weaken). This is summarized by the following procedure:

```
Procedure OpticalFlow(I_1,I_2,J)
| I_1 and I_2 2 consecutive images of size X × Y
| J the coarsest resolution
for j=-J:0
    for k_1=1:2^{-j}X and k_2=1:2^{-j}Y
        if the system (17) is badly conditioned
            V^j_k = V^{j-1}_{k/2}
        else Sol = (M^j_k)^{-1} Y^j_k
            if ||Sol|| > M2^j
                V^j_k = V^{j+1}_{k/2}
            else V^j_k = Sol
            endif
        endif
    endfor
endfor
```

5 The choice of wavelets

In image processing we use a tensorial product of a scaling function ϕ the wavelet associated ψ in $L^2(\mathbf{R})$ to define a basis of $L^2(\mathbf{R}^2)$ with 3 functions:

$$
\begin{aligned}
\Psi^1(x, y) &= \phi(x)\psi(y) \\
\Psi^2(x, y) &= \psi(x)\phi(y) \\
\Psi^3(x, y) &= \psi(x)\psi(y)
\end{aligned} \tag{18}
$$

In our case to solve "the aperture problem", we must choose complex valued wavelets. Indeed as we seek a real solution, if the equations (10) are complex, the system is equivalent to:

$$\begin{bmatrix} ReM_u \\ ImM_u \end{bmatrix} \quad V = \begin{bmatrix} ReX_u \\ ImX_u \end{bmatrix}. \tag{19}$$

Thus we get 6 equations and 6 unknowns (a, b, c, d, e, f). Solving (19) is then a simple inversion of size 6×6 matrix. We chose the complex Daubechies wavelets [7] of order 4. These wavelets are complex valued, real and imaginary parts featuring symmetries and have 5 vanishing moments. The scaling function and the wavelet are defined as follow:

$$\widehat{\phi}(\xi) = \prod_{j=1}^{\infty} m_0(\frac{\xi}{2^j}) \tag{20}$$

$$\widehat{\psi}(\xi) = m_1(\frac{\xi}{2})\widehat{\phi}(\frac{\xi}{2}) \tag{21}$$

where

$$m_0(\xi) = \sum_{k=-4}^{5} a_k e^{-ik\xi} \tag{22}$$

$$m_1(\xi) = \sum_{k=-4}^{5} (-1)^k \overline{a}_{1-k} e^{-ik\xi} \tag{23}$$

and where the coefficients $\sqrt{2}a_k$ are worth:

k	$\sqrt{2}a_k$
1	0.643003+0.182852i
2	0.151379-0.094223i
3	-0.080639-0.117947i
4	-0.017128+0.008728i
5	0.010492+0.02059i

With this decomposition we can compute the products $\langle f, \Psi^n_{jk}\rangle$ using the fast algorithm of Mallat [9].

6 Experiments and results

We applied here our method to estimate the optical flow in four omnidirectional sequences. Two sequences (fig 1, fig 2) where the camera undergoes an uniform translation motion and two sequences (fig 3, fig 4) where the robot turns on itself. In each case we have to compare our results with the method proposed by Bernard [2]. This method is also based on a projection of the Brightness Change Constraint Equation (1) on a basis wavelets but it supposes the flow is locally constant on the support of this one. We will see that this hypothesis is not convenient in the case of the omnidirectional images. Notice that we do not compute here the error of the optical flow estimation because we do not know the real flow on these sequences. However, the various figures make it possible to see the improvement made by the new assumption.

In the case of an uniform translational motion of the omnidirectional camera, the method of Bernard [2] remains satisfactory with a small motion (fig 1(b)) to consider way total the flow on the image. Indeed to say the flow is constant can be a sufficient assumption on an area of the image where the flow is vertical (left center and right center of the image). However this method shows already gaps with a large motion (fig 2(b)) because the distortion due to the sensor is not negligible in this case. Our method (fig 1(a), fig 2(a)) makes possible to correct these problems thanks to a weaker hypothesis on the zone where we calculate the flow.

In the case of a rotational motion, the hypothesis used by Bernard [2], Lucas and Kanade [8] and others in the case of the plane images are never valid. Even if this assumption can be a good approximation for a small rotation the flow obtained with our method (3(a)) is visually better than with the method proposed by Bernard (3(b)). In the case of a large rotational, the assumption of flow locally constant cannot be used (4(b)). By contrast our method always gives good results because the hypothesis (7) takes account of the rotational motions.

7. Conclusions and future works

In this paper we have described a method to estimate the optical flow on the omnidirectional images. This method computes the flow in image sequences starting from the Brightness Change Constraint Equation. To solve "the aperture problem" we have to project this equation on a basis of wavelets. Then we made a hypothesis of affinity of the flow in the neighborhood of the points where velocity is computed. This hypothesis is weaker than the hypothesis generally made in the case of planar images and allows to estimate the flow more accurately. Moreover the use of discrete wavelet basis allows to compute the optical flow with good speed. Indeed computations were carried out on a PC PENTIUM 933 MHz with an implementation under MATLAB and the optical flow computation on 5 resolutions for a monochromic image of size 256×256 is about 20s (we can show that our algorithm have a complexity $\mathcal{O}(N^2)$ for a image of size $N \times N$) . This is a significant point for a real time application.

However our choice of hypothesis even if it makes possible to obtain good results does not characterize correctly the non-linear distortions of the images due to the mirror geometry. It would be interesting in our future works to modelize displacements of the points in the image according to the studied sensor and thus to obtain a hypothesis on the flow which cope with the distortions.

Acknowledgments

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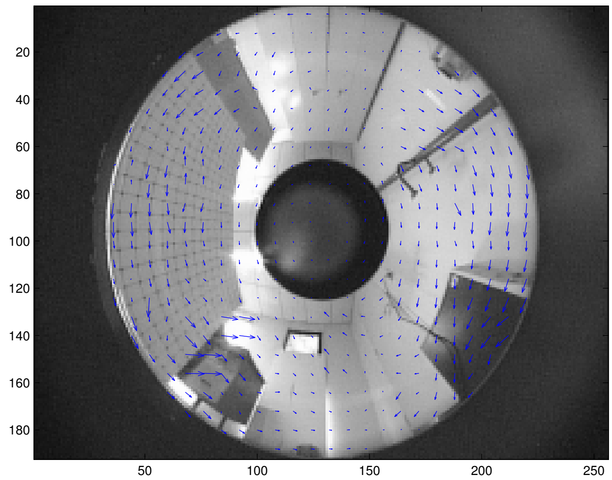
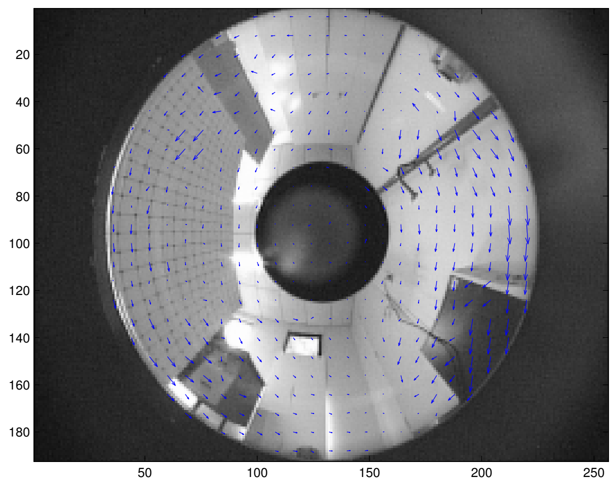
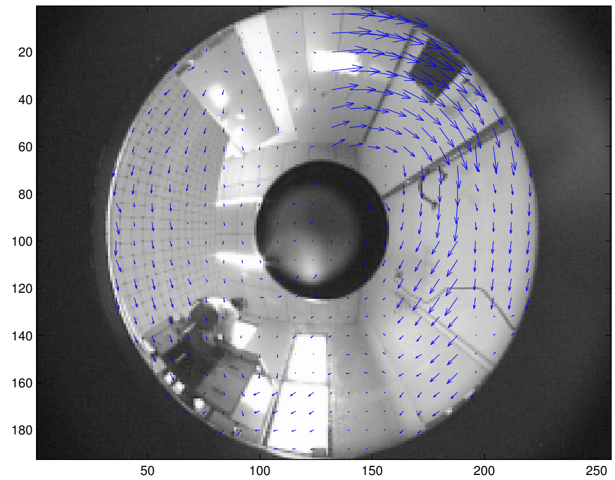


(a)

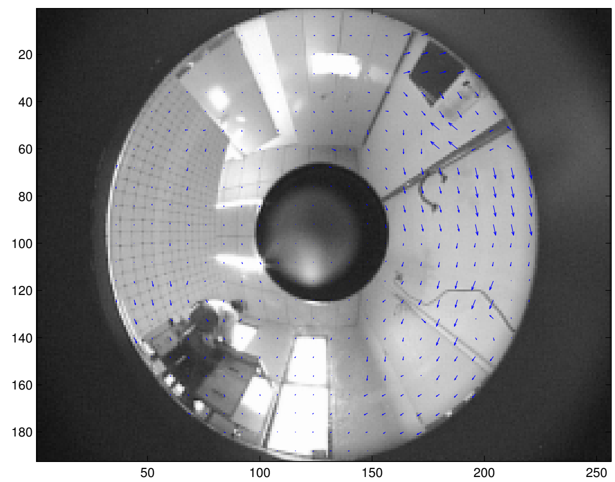


(b)

Figure 1: Omni-images taken from a small translational motion (a) optical flow estimation with our method, (b) optical flow estimation with method proposed by Bernard.

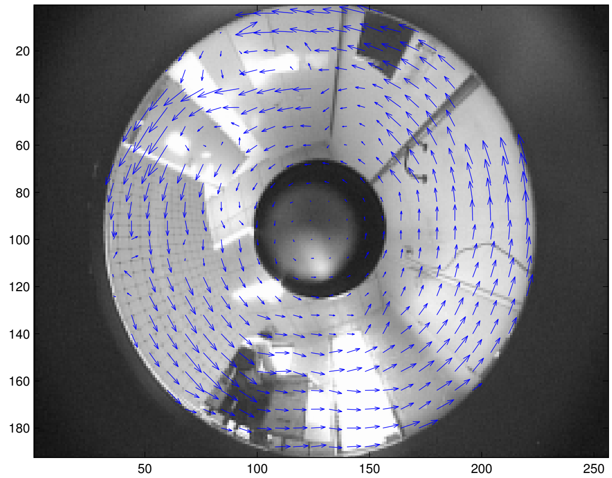


(a)

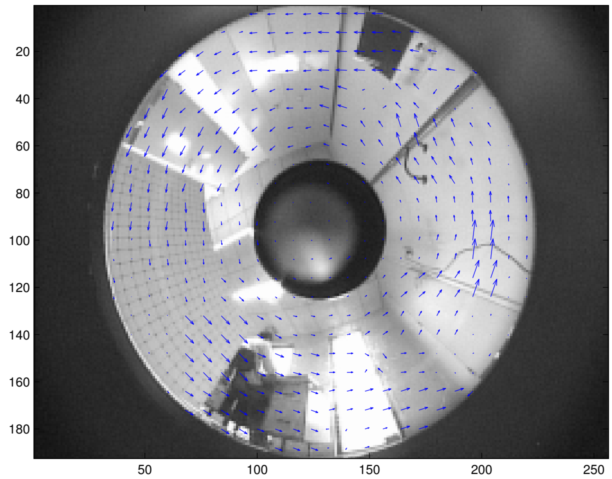


(b)

Figure 2: Omni-images taken from a large translational motion (a) optical flow estimation with our method, (b) optical flow estimation with method proposed by Bernard.

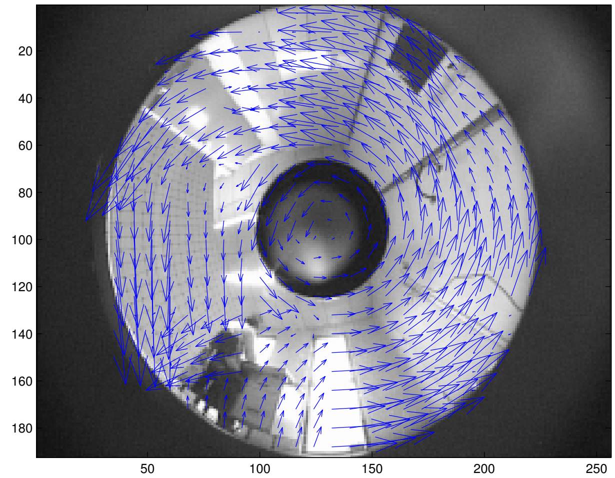


(a)

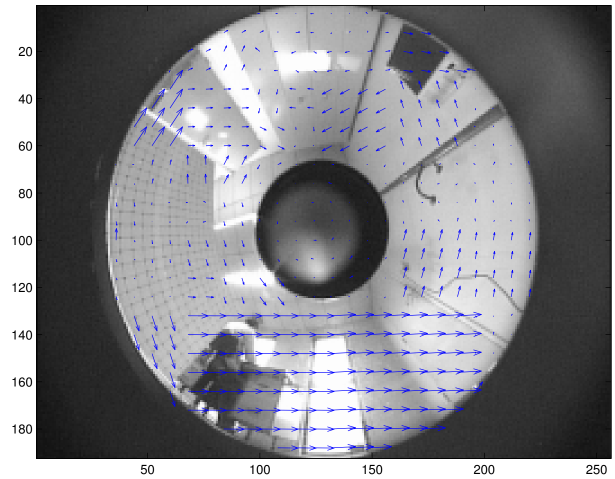


(b)

Figure 3: Omni-images taken from a small rotational motion (a) optical flow estimation with our method, (b) optical flow estimation with method proposed by Bernard.



(a)



(b)

Figure 4: Omni-images taken from a large rotational motion (a) optical flow estimation with our method, (b) optical flow estimation with method proposed by Bernard.