Name:

Exam 1, CS 201: There are five problems on this test. You do not need to calculate final numbers, answers in the form of $15! + 2^4$ are acceptable. Partial credit depends on clear explanations of your work. Make sure you proofread your answers. Good Luck.

Facts which are true but may or may not be relevant:

- Program Correctness Proofs have 6 parts. (1) State loop invariant. (2) Prove loop invariant is true before the loop. (3) Prove loop invariant is true after an arbitrary iteration of the loop. (4) Prove the loop invariant and the loop stopping condition imply the final assertion. (5) Prove that the loop terminates.
- $\lfloor x \rfloor$ is the largest integer less than or equal to x.
- Q is the set of rational numbers, Z is the set of integers, R is the set of real numbers.
- $sign(x) = 1$ if $x \geq 0$, $sign(x) = -1$ if $x < 0$
- $|x| = x$ if $x \geq 0$, $|x| = -x$ if $x < 0$
- Some induction proofs require more than one base case.
- Given sets A,B, an item is in the set $A - B$ if and only if (it is in set A and it is not in set B).
- Given sets A,B, an item is in the set $A \cup B$ if and only if (it is in set A or it is in set B).
- Two sets A,B are equal when an element x is in A if and only if x is in B.

(for graders use:)

1	2	3	4	5	Total

1) (8 points) For this problem, a poker hand is DEFINED to be an unordered set of 5 cards, drawn (without replacement) from the 52 total cards in a deck (13 ranks, and 4 suits).

(a) A straight is any poker hand that has 5 consecutive numbers, for instance, the 8, 9, 10, Jack, Queen, but they can be different suits. How many possible pokers hands are a "straight"?

(b) A flush is a poker hand where every card has the same suit for instance the 2 of clubs, 4 of clubs, 8 of clubs, Jack of clubs, Ace of clubs. How many possible poker hands are a "flush"?

2) (8 points) For each function f defined below, say whether f is (a) 1-1, and (b) onto. When f is not one of those, argue, briefly, why. Pay attention to the domain and co-domain specified.

(a) $f : R \longrightarrow Z, f(x) = \lfloor 2x \rfloor$

- 1-1?
- onto?

(b) $f : Z \longrightarrow Z, f(x) = 2^x - x^2$

- 1-1?
- onto?

(c) $f : Z \times Z \longrightarrow Q, f(x, y) = \frac{x}{y}$

- 1-1?
- onto?

(d) $f : Z \longrightarrow R, f(x) = \frac{1}{x+0.5}$

- 1-1?
- onto?

3) (8 points) Use a loop invariant to prove the correctness of the following algorithm to compute the maximum of a sequence of integers $a_0, a_1, a_2, \ldots, a_{n-1}$ with respect to the initial assertion $(n > 0)$ and the final assertion that $ans = max(a_0, a_1, a_2, \ldots, a_{n-1})$.

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procedure computeMax(a_0, a_1, a_2, ..., a_{n-1})

  ans = a_0;
  i = 1;
  while (i < n) do
      if a_i > ans then
          ans = a_i
      fi
      i = i + 1
  od
  return ans
```

4) (8 points)

Define a function g(n) for integer $n \geq 0$ by the following recurrence:

$g(0) = 3$
$g(1) = 4$
for $n \geq 2$, $g(n) = 4g(n-1) - 4g(n-2)$.
(a, 2 points) Calculate: g(2), g(3), g(4), g(5)

(b, 6 points) Prove that $g(n) = (3-n)2^n$ for $n \geq 0$.

5) (5 points) Prove the following identity is true for sets A,B:

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$