1. You are given two sets $A$ and $B$ of points in the plane, both of size $n$. Recall that a Ham-Sandwich cut is a line that bisects both point sets, such that there are at least $\lfloor n/2 \rfloor$ points of $A$ and of $B$ on each side of the cut. We define a connection between $A$ and $B$ as a set of $n$ line segments, each connecting a point of $A$ to some point of $B$ (i.e., the segments define a one-to-one correspondence between $A$ and $B$). The connection is disjoint if no two segments intersect.

Assuming you are given an algorithm for computing the Ham-Sandwich cut in $O(n \log n)$ time. Give an $O(n \log^2 n)$-time algorithm that outputs a disjoint connection between $A$ and $B$.

2. Let $L$ denote a set of $n$ lines in the plane and let $A(L)$ denote their arrangement. Let $S$ be the set of vertices in $A(L)$. Give an $O(n \log n)$ time algorithm that computes $CH(S)$, and prove that it is correct. Note that $S$ has size $O(n^2)$ because it is the intersections between every pair of $n$-lines, so your algorithm does not have time to explicitly compute all elements of $S$. The figure below shows an arrangement of 6 lines, the 15 intersection points and the convex hull that you need to compute. Note that points may not be in general position, but you only need to return the extreme points along an edge.

3. Consider a collection of $n$ points $P$ in the plane. Define a 3-slab to be the region bounded by a pair of non-vertical parallel lines, such that there are at least 3 points in the region (including on the lines). Define the height of a 3-slab to be the vertical distance between its two lines ($h$ in the picture below). Present an $O(n^2)$-time algorithm which computes a 3-slab of minimum height.