CSE 441T/541T Advanced Algorithms **Fall 2008**

Homework 4a: More NP-Completeness, and Approximation

Assigned: October 29, 2008 *Due Date: November 5, 2008*

For NP-completness reductions, unless otherwise indicated, you should reduce from one of: SAT, 3-SAT, SUBSET-SUM, PARTITION, CLIQUE, INDEPENDENT SET, or VERTEX COVER. You must submit your homework with a signed cover sheet attached to the front.

Core Problem

1. (15 pts) In class, we showed that INDEPENDENT-SET $\leq_p$ VERTEX-COVER. Recall that a graph G with n vertices has an independent set of size at least k iff it has a vertex cover of size at most $n - k$; in particular, the vertices *not* in the cover form an independent set.

   In class, we saw a 2-approximation algorithm A for finding the minimal vertex cover. Professor Ptolemy suggests that one can derive a constant-factor approximation algorithm for independent set by first reducing it to a vertex cover problem as above, then applying algorithm A. Is the professor correct? Justify your answer, either by proving that there exists a constant c for which the above approach gives a c-approximation, or by showing that for any constant c, there exist graphs G (with arbitrarily many vertices) for which the algorithm is *not* a c approximation.

Advanced Problem

This problem is *required* only for CSE 541 students. 441 students may receive extra credit for a correct solution.

2. (10 pts) Consider the following problem. You are given a simple undirected weighted graph G and an integer $f \leq |V|$. Your goal is to pick the location for f vertices of G as casino locations so that the *length of the shortest path* between any vertex in G and its closest casino is the minimum possible. From this optimization problem we create the following decision problem that we will call the CASINO-PLACEMENT problem:

   Given an undirected weighted graph G, an integer $f \leq |V|$, and a real number $d \geq 0$, is it possible to select f vertices of G as casino locations so that the length of the shortest path between any vertex in G and its nearest casino is at most d.

   You are to do **ONE** of the following two tasks:

   - Prove that there is a polynomial time algorithm for the CASINO-PLACEMENT problem. (In other words give an algorithm that you can prove will give the correct answer and that runs in polynomial time.)
   - Prove that CASINO-PLACEMENT is NP-complete.