Practice Problems:

1. Consider the following English arguments. Define propositions/predicates and translate these arguments into logic, then prove or disprove whether the form of the argument is valid.

   (a) All Computer Science majors are people.
   Some computer science majors are logical thinkers

   Some people are logical thinkers.

   (b) If I like mathematics then I will study.
   Either I don’t study or I pass mathematics
   If I don’t pass mathematics, then I don’t graduate.

   If I graduate, then I like mathematics.

2. Using (and citing) rules of inference and logical equivalence, prove the following form of argument is valid:

   \[ P \lor Q \]
   \[ P \rightarrow R \]
   \[ Q \rightarrow R \]

   \[ R \]

3. Using (and citing) rules of inference and logical equivalence, prove the following form of argument is valid:

   \[ (P \land Q) \lor R \]
   \[ R \rightarrow S \]

   \[ P \lor S \]

4. A real number \( x \) is an upper bound of a set \( S \) of real numbers if \( x \) is greater than or equal to every number of \( S \).

   (a) Use quantifiers to express the fact that \( x \) is an upper bound of \( S \). That is, define the proposition \( UB(x) \) that is true when \( x \) is an upper bound of \( S \).

   (b) New definition: A real number \( x \) is called the least upper bound of a set \( S \) of real numbers if \( x \) is an upper bound of \( S \), and \( x \) is less than or equal to every upper bound of \( S \).

   Use quantifiers to express the fact that \( x \) is a least upper bound of \( S \).

5. Write the logical negation of the following quantified statement, and express your answer in a form where the negation sign is inside the scope of all the quantifiers.

   \[ \forall x P(x) \lor \neg (\forall y Q(y)) \]
Problems to turn in

1. Using (and citing) rules of inference and logical equivalence, Prove the following form of argument is valid:

\[
\begin{align*}
& P \rightarrow (Q \lor R) \\
& R \rightarrow S \\
& \neg S \rightarrow \neg Q \\
& P \\
& \hline
& S
\end{align*}
\]

SOLUTION:

score rubric. Each problem worth 5 points total. need both valid steps and rules. Steps combining "simple rules" such as double negation and deMorgan’s are ok.

\[
\begin{align*}
1. & P \rightarrow (Q \lor R)\quad & \text{given} \\
2. & R \rightarrow S\quad & \text{given} \\
3. & \neg S \rightarrow \neg Q\quad & \text{given} \\
4. & P\quad & \text{given} \\
5. & Q \rightarrow S\quad & \text{contrapositive, 3} \\
6. & \neg Q \lor S\quad & \text{ImpliesRule, 5} \\
7. & \neg R \lor S\quad & \text{ImpliesRule, 2} \\
8. & (\neg Q \lor S) \land (\neg R \lor S),\quad & \text{addition, 6, 7} \\
9. & (\neg Q \land \neg R) \lor S,\quad & \text{addition} \\
10. & \neg (Q \lor R) \lor S,\quad & \text{DeMorgans} \\
11. & (Q \lor R) \rightarrow S,\quad & \text{ImpliesRule} \\
12. & P \rightarrow S\quad & \text{hypotheticalsyllogism, 1, 11} \\
13. & S\quad & \text{ModusPonens, 4, 12}
\end{align*}
\]

2. Using (and citing) rules of inference and logical equivalence, Prove the following form of argument is valid:

\[
\begin{align*}
\neg P(a) \rightarrow Q(a) \\
P(a) \rightarrow Q(a) \\
\forall x, Q(x) \rightarrow S(x) \\
\hline
S(a)
\end{align*}
\]

SOLUTION:
3. Write the logical negation of the following quantified statement, and express your answer in a form where the negation sign is inside the scope of all the quantifiers. Use logical equivalences to prove that your formula is equivalent

$$\forall x P(x) \rightarrow \forall y (Q(y) \land R(y))$$

**SOLUTION:**

By the rules of the problem, the solution could stop at step 4 or go on to step 5

1. $$\neg(\forall x P(x) \rightarrow \forall y (Q(y) \land R(y)))$$  
   start
2. $$\neg(\forall x P(x) \lor \forall y (Q(y) \land R(y)))$$  
   1, ImpliesRule
3. $$\forall x P(x) \land \neg\forall y (Q(y) \land R(y)))$$  
   2, deMorgan
4. $$\forall x P(x) \land \exists y \neg (Q(y) \land R(y)))$$  
   3, deMorgan
5. $$\forall x P(x) \land \exists y (\neg Q(y) \lor \neg R(y)))$$  
   4, deMorgan