1A. \[ A = \{1, 2, 3, 4\} \] Relation R is \[ \{ (1, 1), (1, 3), (1, 4), (3, 2), (2, 4), (4, 4) \} \]

transitive closure includes all ordered pairs in R, and
\[ (1, 2), (1, 1), (2, 2), (3, 3), (3, 1) \]

b. prob. 8 bit bit string has more 1's than 0's

\[
\frac{\text{num of bitstrings w/ 5 1's} + \text{num w/ 6 1's} + \text{num w/ 7 1's} + \text{num w/ 8 1's}}{2^8}
\]

\[
= \frac{\binom{8}{5} + \binom{8}{6} + \binom{8}{7} + \binom{8}{8}}{2^8}
\]

(4. \(\binom{13}{3}\) - pick 3 cards of that suit)

(52\(\binom{3}{3}\) - pick 3 cards)

c. TRUE

d. i. prob. that 3 cards are same suit:

\[
\frac{\binom{10}{2}}{\binom{49}{3}}
\]

d. ii. prob. that next 2 cards are same suit:

\[
\frac{\binom{10}{2} \binom{10}{2} \binom{10}{2}}{\binom{49}{3} \binom{49}{3} \binom{49}{3}}
\]

2. (a) 3 ordered pairs in relation
\[ (1, 2), (1, 3), (1, 4), (1, 5), (2, 4) \]

(b) Prove R is equiv. Relation

Prove R reflexive.

Let \( x \) be any integer.

\[ \frac{x}{x} = 1 \] so \( x \) is rational

So \((x, x) \in R\)

So R is reflexive.

Prove R symmetric.

Let \( x, y \) be any 2 numbers.

If \((x, y) \in R\) then integers

\[ x = \frac{a}{b} \] rational. So \( x = \frac{a}{b} \)

So \( y = \frac{b}{x} \) so \( y \) is rational

So \((y, x) \in R\) so R is symmetric.

Prove R transitive.

Let \( x, y, z \) be real numbers

Assume \((x, y) \in R \text{ and } (y, z) \in R\)

So \( x = \frac{a}{b} \) and \( y = \frac{c}{d} \)

When \( a, b, c, d \) are rational integers.

\[ \frac{x}{y} = \frac{a}{c} = \frac{ac}{bd} \]

So \( \frac{x}{y} \) is rational

So \((x, z) \in R\)

So R is transitive.
3a. \[ \text{Prob of pair of kings or queens in 2 cards} \]
\[ \frac{\binom{4}{2} + \binom{4}{2}}{\binom{52}{2}} = \frac{\text{ways to pick 2 kings}}{\text{ways to pick 2 cards}} \]

3b. Full house after starting w/ pair of queens
\[ \frac{12 \cdot \binom{4}{3} + 2 \cdot (12) \cdot \binom{4}{2}}{\binom{50}{3}} \]
\[ \text{pick another rank, then pick 3 cards} \quad \text{or} \quad \text{pick 1 more queen, then pick} \]
\[ \text{another rank and 2 cards of that rank.} \]

4. If G has a path of length \( n+1 \) or longer, then that path must visit the same vertex \( v \) more than once (by the pigeonhole principle). Therefore some sequence of edges starts at a node \( v \) and comes back to that node; this is a cycle.

5. (a) accept binary strings that are \# of form \( 3k+2 \)

(b) end in 110

\[ \text{C has 3 zeros or an even \# of 1's.} \]

\[ \text{(d) contains 1010} \]