

The π -Calculus

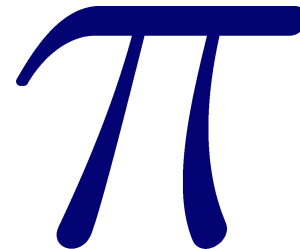
(A Crash Course)

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Syntax

$$P ::= 0 \mid P + P \mid \tau.P \mid a.A \mid \bar{a}.C \mid P \mid P \mid$$

$$\text{if } a = b \text{ then } P \text{ else } P \mid \underbrace{\nu a.P}_{\text{(binds } a \text{ in } P)} \mid D(\tilde{a})$$

(binds a in A)

$$A ::= \underbrace{(a)A}_{\text{(binds } a \text{ in } A)} \mid P$$

$$C ::= \langle a \rangle C \mid \underbrace{\nu a.C}_{\text{(binds } a \text{ in } C)} \mid P$$

a, b, c are names; \tilde{a} is a vector a_1, \dots, a_n ; τ is internal transition

$$a(b_1, \dots, b_n).P \stackrel{\Delta}{=} a.(b_1) \cdots (b_n)P$$

$$\nu b_1, \dots, b_n. \bar{a} \langle c_1, \dots, c_m \rangle.P \stackrel{\Delta}{=} \nu b_1 \cdots \nu b_n. \bar{a}. \langle c_1 \rangle \cdots \langle c_m \rangle P$$

* Syntax from [Dam]

Transition Rules

$$\begin{array}{l}
 \text{SUM} \quad \frac{P \xrightarrow{\alpha} P'}{P+Q \xrightarrow{\alpha} P'} \qquad \text{PRE} \quad \frac{\cdot}{\alpha.P \xrightarrow{\alpha} P} \\
 \text{COM} \quad \frac{P \xrightarrow{\nu \tilde{b}. \tilde{a} \langle \tilde{b}' \rangle} P' \quad Q \xrightarrow{a(\tilde{c})} Q'}{P \mid Q \xrightarrow{\tau} \nu b.(P' \mid (Q' \{ \tilde{b}' / \tilde{c} \}))} \qquad \text{PAR} \quad \frac{P \xrightarrow{\alpha} P'}{P \mid Q \xrightarrow{\alpha} P' \mid Q} \\
 \text{IF}_1 \quad \frac{P \xrightarrow{\alpha} P'}{\text{if } a = a \text{ then } P \text{ else } Q \xrightarrow{\alpha} P'} \\
 \text{IF}_2 \quad \frac{Q \xrightarrow{\alpha} Q'}{\text{if } a = b \text{ then } P \text{ else } Q \xrightarrow{\alpha} Q'} \quad (a \neq b) \\
 \text{RES} \quad \frac{P \xrightarrow{\alpha} P'}{\nu a.P \xrightarrow{\alpha} \nu a.P'} \quad (a \notin \text{names}(\alpha)) \\
 \text{OPEN} \quad \frac{P \xrightarrow{\nu \tilde{b}. \tilde{a} \langle \tilde{b}' \rangle} P'}{\nu c.P \xrightarrow{\nu \tilde{b}. c. \tilde{a} \langle \tilde{b}' \rangle} P'} \quad (c \neq a, c \in \tilde{b}' \setminus \tilde{b}) \\
 \text{ID} \quad \frac{P \{ \tilde{b} / \tilde{a} \} \xrightarrow{\alpha} P'}{D(\tilde{b}) \xrightarrow{\alpha} P'} \quad (D(\tilde{a}) \triangleq P)
 \end{array}$$

* Transition rules from [Dam]

Informally Speaking...

$a.P$	transition a leaves you at P
$P + Q$	choice
$a.P + \bar{a}.Q$	reaction
$P \mid Q$	concurrency
if ϕ then P else Q	conditional
$\nu a.P$	a is bound as a private name
$A(b)$	invocation
$A\langle b \rangle$	transmission

Semantics... by example!

$$\mathbf{Coroutine}(a) = (\nu b).\bar{a}\langle b\rangle.b(c).\mathbf{Coroutine}(c)$$

$$\mathbf{Buf}_1(i, o) = i(a).\bar{o}\langle a\rangle.\mathbf{Buf}_1(i, o)$$

$$\mathbf{Buf}_{n_1+n_2}(i, o) = \nu m.(\mathbf{Buf}_{n_1}(i, m) \mid \mathbf{Buf}_{n_2}(m, o))$$

$$\mathbf{Buf}(i, o) = i(a).\nu m.(\mathbf{Buf}(i, m) \mid \bar{o}\langle a\rangle.\mathbf{Buf}(m, o))$$

$$\mathbf{BufCell}(o, d, n_l, n_r) = \bar{o}\langle d\rangle.\bar{n}_l\langle o, n_r\rangle.0 + n_r(o', n).\mathbf{BufCell}(o', d, n_l, n)$$

$$\begin{aligned} \mathbf{StartCell}(i, o, n) = & i(d).\nu o'.\nu n'.(\mathbf{StartCell}(i, o', n') \mid \\ & \mathbf{BufCell}(o, d, n', n)) \\ & + n(o', n').\mathbf{StartCell}(i, o', n') \end{aligned}$$

$$\mathbf{GCBuf}(i, o) = \nu n.\mathbf{StartCell}(i, o, n)$$

* Examples from [Dam]

Dam's π - μ -Calculus

- A first-order modal μ -calculus with π 's communication features
 - connectives $\wedge, \vee, \leftarrow, \rightarrow, \nu \leftarrow, \nu \rightarrow$
 - assertions
 - validity, satisfiability
 - cuts
 - closed and open correctness
 - basic judgments
 - sound, weakly complete proof system

References

- [**Dam**] Dam, Mads. Proof systems for pi-calculus logics. To appear in de Queiroz (ed.), *Logic for Concurrency and Synchronisation*, Studies in Logic and Computation, Oxford Univ Press, 2003(?).
- [**Lin 1994**] Lin, H. Symbolic bisimulations and proof systems for the pi-calculus. Technical Report 1994:07, COGS, University of Sussex, 1994.
- [**Milner 1999**] Milner, Robin. *Communicating and mobile systems: the π -calculus*. Cambridge University Press, 1999.

Discussion

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