The π-Calculus

(A Crash Course)

Morgan Deters

mdeters@cs.wustl.edu

Washington University in St. Louis

π

`www.cs.wustl.edu/~doc/`

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Syntax

$$P \quad ::= \quad 0 \;\Big|\; P + P \;\Big|\; \tau.P \;\Big|\; a.A \;\Big|\; \bar{a}.C \;\Big|\; P|P \;\Big|$$

$$\textbf{if } a = b \textbf{ then } P \textbf{ else } P \;\Big|\; \underbrace{\nu a.P}_{\textbf{(binds } a \textbf{ in } P\textbf{)}} \;\Big|\; D(\tilde{a})$$

$$A \quad ::= \quad \overbrace{(a)A}^{\textbf{(binds } a \textbf{ in } A\textbf{)}} \;\Big|\; P$$

$$C \quad ::= \quad \langle a \rangle C \;\Big|\; \underbrace{\nu a.C}_{\textbf{(binds } a \textbf{ in } C\textbf{)}} \;\Big|\; P$$

a, b, c **are *names*; $\tilde{a}$ is a *vector* $a_1, \ldots, a_n$; τ is *internal transition***

$$a(b_1, \ldots, b_n).P \quad \triangleq \quad a.(b_1) \cdots (b_n)P$$

$$\nu b_1, \ldots, b_n.\bar{a}\langle c_1, \ldots, c_m \rangle.P \quad \triangleq \quad \nu b_1. \cdots .\nu b_n.\bar{a}.\langle c_1 \rangle \cdots \langle c_m \rangle P$$

*** Syntax from [Dam]**

Transition Rules

Sum $$\frac{P \xrightarrow{\alpha} P'}{P+Q \xrightarrow{\alpha} P'}$$

Pre $$\frac{\cdot}{\alpha.P \xrightarrow{\alpha} P}$$

Com $$\frac{P \xrightarrow{\nu\tilde{b}.\bar{a}\langle\tilde{b'}\rangle} P' \quad Q \xrightarrow{a(\tilde{c})} Q'}{P \mid Q \xrightarrow{\tau} \nu b.(P' \mid (Q'\{\tilde{b'}/\tilde{c}\}))}$$

Par $$\frac{P \xrightarrow{\alpha} P'}{P \mid Q \xrightarrow{\alpha} P' \mid Q}$$

If$_1$ $$\frac{P \xrightarrow{\alpha} P'}{\textbf{if } a = a \textbf{ then } P \textbf{ else } Q \xrightarrow{\alpha} P'}$$

If$_2$ $$\frac{Q \xrightarrow{\alpha} Q'}{\textbf{if } a = b \textbf{ then } P \textbf{ else } Q \xrightarrow{\alpha} Q'} \quad (a \neq b)$$

Res $$\frac{P \xrightarrow{\alpha} P'}{\nu a.P \xrightarrow{\alpha} \nu a.P'} \quad (a \notin \textbf{\textit{names}}(\alpha))$$

Open $$\frac{P \xrightarrow{\nu\tilde{b}.\bar{a}\langle\tilde{b'}\rangle} P'}{\nu c.P \xrightarrow{\nu\tilde{b}.c.\bar{a}\langle\tilde{b'}\rangle} P'} \quad (c \neq a,\ c \in \tilde{b'} \backslash \tilde{b})$$

Id $$\frac{P\{\tilde{b}/\tilde{a}\} \xrightarrow{\alpha} P'}{D(\tilde{b}) \xrightarrow{\alpha} P'} \quad (D(\tilde{a}) \triangleq P)$$

*** Transition rules from [Dam]**

Informally Speaking...

$a.P$	**transition** a **leaves you at** P
$P + Q$	**choice**
$a.P + \bar{a}.Q$	**reaction**
$P \mid Q$	**concurrency**
if ϕ **then** P **else** Q	**conditional**
$\nu a.P$	a **is bound as a private name**
$A(b)$	**invocation**
$A\langle b\rangle$	**transmission**

Morgan Deters ⟨mdeters@cse.wustl.edu⟩

Semantics... by example!

$$\textbf{\textit{Coroutine}}(a) = (\nu b).\bar{a}\langle b\rangle.b(c).\textbf{\textit{Coroutine}}(c)$$

$$\textbf{\textit{Buf}}_1(i,o) = i(a).\bar{o}\langle a\rangle.\textbf{\textit{Buf}}_1(i,o)$$

$$\textbf{\textit{Buf}}_{n_1+n_2}(i,o) = \nu m.(\textbf{\textit{Buf}}_{n_1}(i,m) \mid \textbf{\textit{Buf}}_{n_2}(m,o))$$

$$\textbf{\textit{Buf}}(i,o) = i(a).\nu m.(\textbf{\textit{Buf}}(i,m) \mid \bar{o}\langle a\rangle.\textbf{\textit{Buf}}(m,o))$$

$$\textbf{\textit{BufCell}}(o,d,n_l,n_r) = \bar{o}\langle d\rangle.\overline{n_l}\langle o,n_r\rangle.0 + n_r(o',n).\textbf{\textit{BufCell}}(o',d,n_l,n)$$

$$\begin{aligned}\textbf{\textit{StartCell}}(i,o,n) = \; & i(d).\nu o'.\nu n'.(\textbf{\textit{StartCell}}(i,o',n') \mid \\ & \qquad \textbf{\textit{BufCell}}(o,d,n',n)) \\ & + n(o',n').\textbf{\textit{StartCell}}(i,o',n')\end{aligned}$$

$$\textbf{\textit{GCBuf}}(i,o) = \nu n.\textbf{\textit{StartCell}}(i,o,n)$$

*** Examples from [Dam]**

Morgan Deters ⟨mdeters@cse.wustl.edu⟩

Dam's π-μ-Calculus

- **A first-order modal μ-calculus with π's communication features**
  - **connectives** $\wedge, \vee, \leftarrow, \rightarrow, \nu \leftarrow, \nu \rightarrow$
  - **assertions**
  - **validity, satisfiability**
  - **cuts**
  - **closed and open correctness**
  - **basic judgments**
  - **sound, weakly complete proof system**

References

[Dam] Dam, Mads. Proof systems for pi-calculus logics. To appear in de Queiroz (ed.), *Logic for Concurrency and Synchronisation*, Studies in Logic and Computation, Oxford Univ Press, 2003(?).

[Lin 1994] Lin, H. Symbolic bisimulations and proof systems for the pi-calculus. Technical Report 1994:07, COGS, University of Sussex, 1994.

[Milner 1999] Milner, Robin. *Communicating and mobile systems: the π-calculus*. Cambridge University Press, 1999.

Discussion

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