

Rationales and Argument Moves

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Abstract. We discuss five kinds of representations of rationales and provide a formal account of how they can alter disputation. The formal model of disputation is derived from recent work in argument. The five kinds of rationales are compression rationales, which can be represented without assuming domain-knowledge (such as utilities) beyond that normally required for argument. The principal thesis is that such rationales can be analyzed in a framework of argument not too different from what AI already has. The result is a formal understanding of rationales, a partial taxonomy, and a foundation for computer programs that represent and reason with rationales.

The five kinds of rationales are as follows: (c)ompression and (s)pecialization, which yield rules, and (d)isputation, which yields a decision. These are modeled as potentially changing the focus of the dispute. Then there are (f)it, a rationale for rules, and (r)esolution, a rationale for decisions. These cannot be modeled as simply; they force disputation to a meta-level, at least temporarily.

The paper first discusses each kind of rationale in the abstract. Then it produces a model of dispute in which the simpler rationales can be analyzed. Formal examples are given. The model is augmented to allow analysis of the more difficult rationales. Examples are again given. The discussion is not intended to be strictly mathematical; rather, it aims to use formal methods to illuminate and provide framework for future interpretation and implementation.

Key words. rationale, ratio legis, ratio decidendi, principle, purpose, dialectic, procedure, argument, rule, policy, backing, defeasible reasoning, case-based reasoning, logic.

I. Rationale.

In disputation, claims are supported by arguments, which refer to rules, cases, and evidence. Sometimes, the rationales of rules and the rationales of decisions in cases appear in disputation as well.

Rationales appear in procedural contexts such as debate, dialectical inquiry, and legal argument. In purely declarative contexts, wherein logic usually is defined, the effect of rationales on inference is more mysterious; there appears to be no important role for rationales of rules, once the rules have been adopted.

A rationale for a rule is a structure that contains relevant information about the reason for the rule's adoption. A rationale for a case, similarly, is relevant additional information about the decision reached. Following Toulmin [1958], one can refer to the rationale as the "backing" of the "warrant." In the language of legal philosophy, *ratio decidendi*¹ and *ratio legis* are rationales. Recent authors have referred to "principle" or "purpose." The essential question is what are the appropriate forms of rationales and how they can be brought to bear in argument.

There are many kinds of rationales; rules are adopted on various grounds. Defeasible rules can be adopted through game-theoretic or inductive reasoning, on the whim of authority, or by arbitrary convention. A defeasible rule that is a policy, which might be a rule in a legal domain, often balances competing interests. Sometimes the particular balance struck is part of the grounds for the rule's adoption. Sometimes the rule strikes a balance in a principled way, but as often it has the right politic or expedience. Sometimes the grounds for adopting a policy are decision-theoretic. Decisions in precedent-setting cases can be made on similar grounds.

The rationales for rules and cases considered here are not intended to exhaust those that can be imagined. They might generally be classified as *compilation rationales*. Compilation rationales are those that compile pieces of past disputations, past lines of argument, past preferences of one argument over another, past projections from cases. They have the advantage of being modeled fully within the vocabulary of a system of argument. Excluded, for instance, are decision-theoretic rationales, which require the introduction of additional information representing utilities. The present aim is to discuss how dialectical moves force reversion to the forms from which rules were compiled, and how existing models of argument might usefully be augmented, how the repertoire of formally modeled moves might be increased.

The compilation rationales to be discussed are as follows. For rules: adopt a rule because it compresses a line of reasoning (compression, or c-rationale); adopt a rule because it specializes a general principle (specialization, or s-rationale); adopt a rule because it expresses a regularity that fits a set of cases (fit, or f-rationale). For cases: form an opinion despite competing factors by resolving the competition (resolution, or r-rationale); form an opinion because the set of arguments in the recorded disputation of the case, which may be incomplete, warrants the opinion (disputation, or d-rationale).

To attack a well-formed argument, with rationales unmentioned, there are usually a couple of alternatives:

1. Literally, *ratio decidendi* is "reasoning(s) of the deciding(s)." Unlike most Latin nouns, neither *ratio* nor *decidendi* changes its ending throughout its declension, since each is an (instance of an) irregular (kind of) gerund. As one referee notes, contemporary common use of *ratio decidendi* is "rule of the case." But that use makes a number of assumptions, e.g., that there *is* a rule of the case, that a rule represents the reasoning of the case, etc., which perhaps are better evaluated once the subject has been studied more carefully in the abstract.

- i. Attack some subpart of the argument or provide a separate argument for a contrary conclusion. If the protocol allows meta-argument, attack the sufficiency of the argument at the present point in the dialogue.
- ii. Attack the legitimacy of the move at the point at which it occurs. That is, show that it does not meet the player's burdens, or provide reasons for claiming that it does not defeat the argument that it is said to defeat, or otherwise impugn its place or station.

There is yet another class of moves, moves that make use of rationales:

- iii. Attack a rule used in the argument by citing its rationale and claiming that the rule's scope is exceeded in this application.
- iv. Or attack a rule that derives from a case, a case-based argument, by citing the rationale for the decision in the case and arguing that the decision is flawed. A new argument that should have been considered has been discovered.
- v. Or argue that the decision provides no guidance in the current situation. An argument that can now be made, in the current situation, would have effectively countered the argument that led to the decision in the case.
- vi. Or argue that a rule is a particular form of a more general rule, or a modern restatement of an older rule, a derivation from broader principle, or a version of a separate rule; then restate the rule in its original form; then attack the new rule, the strength of which has now been altered under considerations of *lex specialis*, *lex posterior*, or *lex superior*

The purpose of this paper is to sketch a formal account of what data would be required to make such moves, and how those moves affect the state of a disputation.

II. Informal Examples of Compilation Rationales.

First, consider each of the rationales in simple examples.

1. *compression*, or *c-rationale*:

As a defeasible rule, "vehicles used for private transportation are not allowed in the park." Meanwhile, defeasibly, "vehicles are normally for private transportation." There is therefore the two-step argument, for disallowing vehicles in the park. Adopt the policy, with the rationale of compression, that as a rule, "no vehicles in the park."

To attack an argument using a rule with a c-rationale, restate the argument in an uncompressed form. The resulting uncompressed argument will be more susceptible to attack. Uncompressed arguments present more points to counterattack. They are also not as direct, so they are more easily defeated in some syntactic accounts of defeat among arguments.

Attack by arguing that emergency vehicles are not used for private transportation. This does not

affect the argument using the compressed rule, but it devastates the uncompressed argument.

Compression tends not to be fragile because rules are typically compressed when the argument being compressed is resistant to counterargument. It would be folly to compress long arguments, for instance, into one-step rules. Arguments constructed thereupon would always be challenged because so many challenges of the uncompressed form will be possible.

2. *specialization, or s-rationale:*

As a principle, “tranquil public spaces should be preserved.” “Parks are tranquil public spaces.” “Disallowing vehicles in a space is a kind of preserving tranquility.” There is currently an argument for a particular park, defeasibly, that it is a public space, and defeasibly, that it should be undisturbed. There is currently no argument that such a park should have vehicles disallowed from being in it. There may be other ways to preserve. The problem is that so far, the converse of the third rule (if we may talk about the converse of a defeasible rule) is missing: “In order to preserve a tranquil space, disallow vehicles.” But a policy-maker might adopt as a rule, “no vehicles in the park,” in order to further the principle in some particular way. When there is a principle served in this way, we call the rule a *rule of implementation*. For any principle, there are many possible competing rules of implementation.

Usually, a rule of implementation is adopted in the presence of counterarguments. For instance, “barring vehicles increases commuting effort.” There are tradeoffs associated with any particular way of implementing a principle. In such instances, the rationale for the rule is more advanced. It is a balance of competing interests, which might actually be an f-rationale, d-rationale, or r-rationale. The s-rationale here is more naive; it is intended to be very simple. One way to preserve is to disallow vehicles; disallowing vehicles implements a kind of preservation. We simply assume that the missing rule is implicit: “defeasibly, preservation entails disallowing vehicles.”

Attack, again, by restating the argument so that the rule in question has its antecedent expressed more generally: “insofar as public spaces should be preserved, no vehicles allowed in public spaces.” A counterargument based on a rule, “vehicles are allowed on public roads in public spaces,” will now suffice as a reply (assuming that the vehicle in question can be argued to have been on a public road). Prior to citing the rationale, such a counterargument might be ineffective because it is less specific: it refers to public spaces, not to parks. Formally, this will be equivalent to attacks on c-rationales, except that there is usually a dominant rule or principle in the argument that is compressed.

Attack, too, by identifying a different way of meeting the demands of the guiding rule, by suggesting a different implementation. This attack corresponds to attacking a rule adopted without full consideration of counterarguments. It will be formally equivalent to attacks using r- or d-rationales.

Since principles can be implemented in numerous ways, rules based on s-rationales are fragile. They eventually will be challenged and replaced with rules or cases with f-, d-, or r-rationales.

3. *fit, or f-rationale:*

Adopt a rule because it expresses a regularity among cases. Cases can be actual or hypothetical, but there should be widespread agreement over the way they are decided.

Rule-adoption based on f-rationale is related to theory-formation in scientific reasoning. As in scientific theory-formation, coherence and simplicity are important. Unlike scientific theory-formation, errors of fit are not tolerable. Though errors are not tolerated, there is a way to eliminate errors of legal fit that is not available to scientific theorizing. Errors of legal fit can be eliminated without altering the offending legal rule. A more specific legal rule that deals with exceptions can simply be added to the body of rules. This option is not possible for scientific theory.

The policy “no vehicles in the park” might allegedly fit the cases:

(case1) *disallowed*: a private automobile driving through the park;

(case2) *disallowed*: an antique automobile parked in the park;

(case3) *disallowed*: a golf cart driven into the park;

(case4) *allowed*: a pedestrian strolling in the park.

An argument using a rule with f-rationale can be attacked by first proposing a new rule that also successfully distinguishes the recorded cases of allowed parking from cases of disallowed parking. Then the attack continues by (a) noting that the new rule no longer applies to the current fact situation (or at least, that the argument that it applies has not been given), or by (b) noting that the reformulated rule applies but is not as specific as originally suggested. In the latter situation, the argument is susceptible to attack by counterarguments that would have been considered less specific on the earlier formulation of the rule. Once again, the defeat relations among arguments can be altered by citing rationale.

Reformulating rules corresponds to arguing over which theory has the best fit. We are unwilling at the moment to postulate the conditions under which a “policy theory” fits better than a competing policy theory. Philosophy of science, similarly, still cannot articulate conditions precisely for one scientific theory fitting better than another. Thus, competing formulations of policies to fit cases merely interfere with each other; neither can defeat the other until coherence and simplicity and other criteria of fit have been addressed.

An argument using a rule with an f-rationale may also be attacked by adding or deleting cases. For example, to attack the rationale of this rule, add a case, such as

(case5) *allowed*: a vehicle delivering a tree to be planted in the park;

or delete a case e.g., claim that (case3) is incorrectly decided. Or propose a different regularity consistent with the cases: “vehicles, excluding those performing essential park or public functions, are not allowed in the park”.

Since it is easy to theorize about cases with sets of defeasible rules that allegedly express regularity, rules with f-rationale will be robust only if the question of fit is addressed in detail.

4. resolution, or r-rationale:

Suppose the case of the hurried government official who detours his automobile through the park raised two arguments that interfered with each other without either defeating the other. “No vehicles in the park.” “Public officials on official business have use of public spaces.” The decision to disallow such detours sets a precedent upon which a variety of future arguments can be constructed. The rationale of the decision, of the precedent, is that it resolves the competing factors: preservation of public space, versus availability of public resources for government business.

To attack an argument relying on this precedent, cite the rationale. Then claim that the current case introduces additional interests which, when weighed, could alter the balance against the prior decision. For instance, suppose a subsequent case also introduces the element of time. In the current case, the official drove through the park when the park was closed. In the prior case, the time of day was Saturday noon. Now arguments based on time of day, government business, and preservation of public spaces must be weighed.

Without citing the rationale and attacking the grounds for the precedent, an argument for allowing the new detour based on time of day could still be made. The argument based on time and the argument based on vehicles in the park are of incomparable specificity. Hence, their disagreement might force a resolution in this case. Discount the old precedent and the new resolution must start anew, weighing time of day and government business against the ban on vehicles in the park. Leave the rationale of prior decision unmentioned, and the current case will compare time of day against an established precedent based both on government business and the ban on vehicles in the park. The difference is subtle.

Because new cases almost invariably introduce significant new factors unweighed in the earlier decision, precedents with r-rationales tend to be fragile. But as noted, the stakes are not high when an r-rationale case is discounted by this kind of attack.

R-rationales are special kinds of d-rationales. So there are other attacks on r-rationales that are described as attacks on d-rationales.

5. disputation, or d-rationale:

Disputation informs the decision in a case. This disputation can be simpler than the weighing of indeterminate factors. A decision might simply be mandated by superior argument, where the adjudicator played no major role. Appeal to such a case in future argument is susceptible to review of the recorded disputation of the case. The arguments that were persuasive in the prior case may not be persuasive in the present case and its new context.

When the record of the disputation that led to decision can be recalled, the decision has a d-rationale. The use of such a precedent depends on whether the result of disputation cannot be significantly corrupted when the same arguments are applied again. Sometimes there are new arguments that can be made that would change the decision or the clarity of the decision. A different attack on a rule with d-rationale occurs when past disputation was incomplete. Not all arguments that could have been advanced were advanced in the prior disputation.

Suppose that in the case of an ambulance parked in the park while its crew was off-duty, the parking was decided to be disallowed. One side argued that the ambulance was permitted to park because it was an emergency vehicle. The adjudicator held, on the other hand, that despite being an emergency vehicle, parking was disallowed for emergency vehicles that were not prepared to respond to an emergency. Suppose the arguments were of such a form that the case could be decided on syntactic grounds. Consider a subsequent case of an off-duty ambulance that remained prepared to respond. The persuasiveness of the precedent depends on whether the decision’s rationale, a d-rationale, is exposed. According to precedent, “off-duty ambulances are not allowed in the park.” The opposition successfully defeats this argument by recalling that an ability to respond to emergency was crucial to the past decision. Imagine that the ambulance’s ability to respond can be established in the present case. The precedent is no longer persuasive.

III. Formal Requirements.

In many models of argument, arguments are produced by chaining rules from a set of defeasible rules, L, and are grounded in a set of undisputed premises, or evidence, E. Case-based reasoning supplements the set of rules by permitting rules to be formulated from cases. There is thus a set, C of cases, the effect of which is to augment L, $Rules(L, C) = L \cup RulesFrom(C)$, given some appropriate way of extracting rules form cases, RulesFrom.

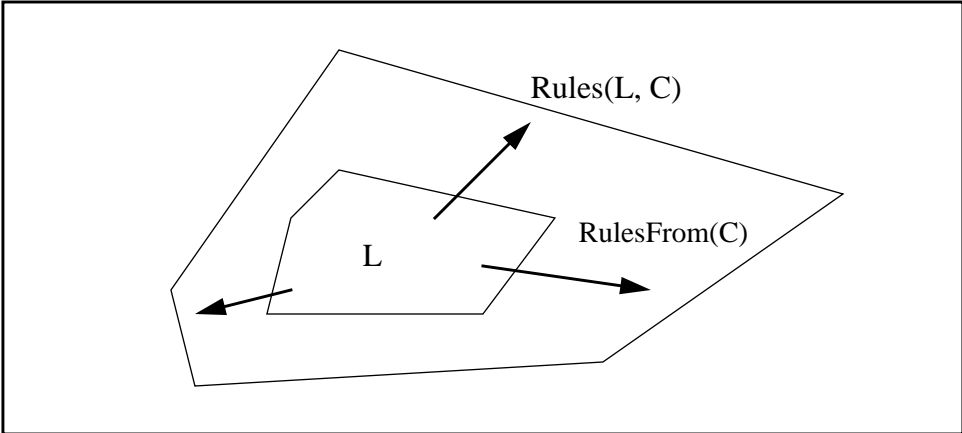


Fig. 1: Cases effectively augment the set of defeasible rules upon which arguments can be built.

This is one framework in which to formalize legal and policy reasoning, a framework inherited from the logicians, suitably altered by researchers in philosophical logic and artificial intelligence. There is modest understanding of the way that cases augment rules in formal systems, as described in the Rissland-Ashley work [1987].

Formally, we may start with a logical language: sentences, S, and a notion of consistency. E is a consistent set of sentences in the logic. L is a set of pairs of sentences, where a pair <p, q> in L means that p is reason for q, and is written $p \succ-- q$. C is a set of case descriptions, where each description is a list of properties (or their negations) holding of the case, and there is a hierarchy among the properties, distinguishing at least between evidence and judgement, and possibly

several levels in between.

One may choose to view the logic as a notation for describing, i.e., for declaring commitments, or as a language in which to conduct discourse according to some procedure, i.e., to compute or discover commitments.

Declaratively (i.e., from a traditional logical point of view), the emphasis is on the complete set of arguments that can be constructed upon L, C, and E: $\text{Args}(L, C, E)$. A conclusion must be warranted with respect to the full complement of arguments: if an effective counterargument is possible, then an argument fails to warrant its conclusion, whether anyone ever finds that counterargument. Alchourron and Bulygin [1971], Soeteman [1989], Simari and Loui [1992] and Prakken [1993] are examples of declarative accounts of argumentative and legal reasoning.

Procedurally (i.e., computationally), parties to a dialogue consume limited resources, producing a succession of sets $\text{Args}_i(L, C, E)$, corresponding to the various stages $i=1, 2, \dots, n$ of the dispute. Warrant is defined with respect to a particular set of arguments at a particular stage, $\text{Args}_i(L, C, E)$. The appropriate stage is determined by a termination rule. Sometimes it is known in advance; sometimes it is a bound that is revealed when it is reached; sometimes an i is appropriate merely because Args_i has a quiescence, e.g., because one party can make no useful response. The protocol for inquiry may allow parties to introduce objects significant to the dispute, other than arguments. For example, challenges might be allowed in the dialogue. Rescher [1997], Alexy [1989], Skalak and Rissland [1991], Loui [1992], Sartor [1993b], Vreeswijk [1993], and Gordon [1993]. See figures 1, 2, and 3.)

The behaviors of the various rationales are sufficiently varied that some complexity is required in order properly to model them.

The c- and s-rationales for rules, and the d-rationale, for cases, are appropriately modeled by substituting less compact arguments (or suites of arguments) for the original argument. Disputation can then continue after the unpacking.

The f-rationale and r-rationale require an entirely different mechanism. When arguments using rules or cases with these rationales are attacked, the dispute is about the outcomes of other disputes. A mechanism for meta-argument is required, and the simplest adequate devices are used here.

We present first the formalism for rationales that can be treated wholly at the object level, then augment the formalism to handle the meta-level disputes. For a formal system to treat all five kinds of compilation rationales, it would have to integrate the mechanisms for unpacking and meta-argument. In the interest of clarity, this is not done here.

III.1. Object-Level Disputation.

Formally, the record of a disputation includes a sequence of moves,

Defn. Disputations as Moves. A disputation is a sequence, $M_n = \langle m_0, m_1, \dots, m_n \rangle$.

Each move may consist of several things: a claim, an argument for the claim, perhaps an argument for the sufficiency of the pair at this point in the disputation.

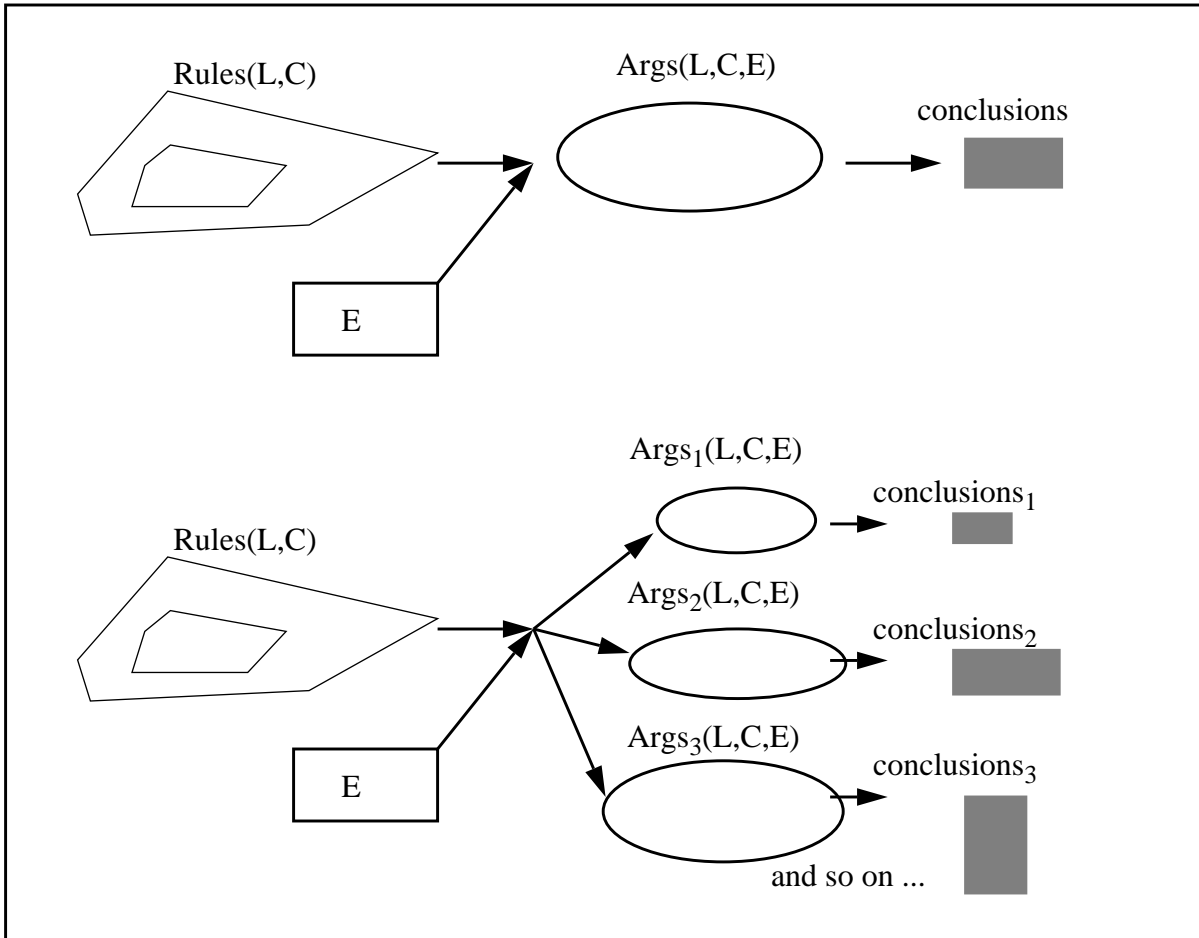


Figure 2. (a) The declarative picture: arguments mediating inference. (b) The procedural picture: arguments constructed through time, with defeasible conclusions: conclusions warranted at a time might be defeated later in disputation.

A simple protocol for dispute is a two-player immediate-response dialectic (several alternate protocols are described in [Loui, 1992]): (

- i. there are two advocates, who are players, and an adjudicator who has no moves;
- ii. moves alternate between players;
- iii. each move must alter current opinion, which is determinable syntactically, and which is either pro, con, or none;
- iv. all arguments must be well-formed: i.e., members of $Args(L, C, E)$;
- v. arguments do not occur repeatedly in the record: i.e., once an argument is introduced, it need not and cannot be introduced again.

The internal structure of the argument, a_i , in each move, m_i , has been the topic of most recent

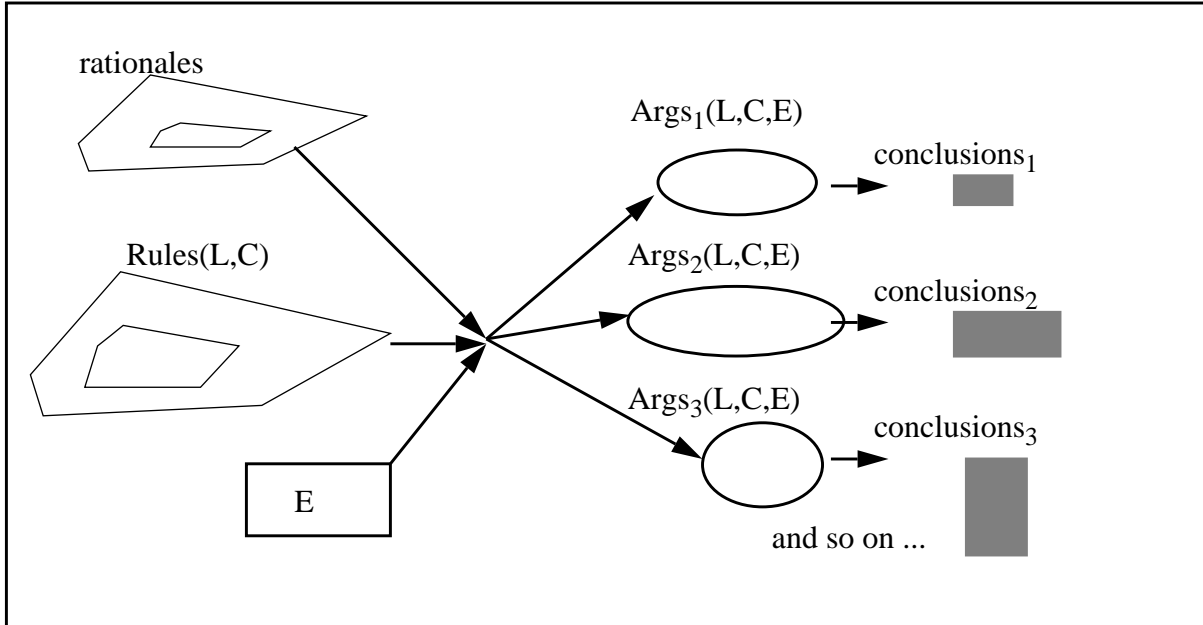


Fig. 3: The picture with rationale-based attacks: arguments are produced and revised through time, producing conclusions. Neither the arguments nor the conclusions grows monotonically.

research on argument (e.g., [Sartor, 1993b], [Prakken, 1993], [Simari & Loui, 1992], [Loui et al., 1993a]). We have intentionally not committed to a particular form of arguments. However, the examples use the following structure for arguments.

Defn. Arguments. An argument is a structure, $\langle \{l_1, l_2, \dots\}, h \rangle$, where each l_j is in $\text{Rules}(L, C)$, and h is the main claim of the argument.

Other authors discuss minimality and connectedness properties at length: that is, how the l_j 's and h relate to one another.

The sufficiency of the move depends crucially on the defeat relations among arguments introduced in M_n .

As an example, the following dispute proceeds according to a reasonable dialectical protocol:

- | | |
|---|---------------|
| 1. pro: $\{e_1 \dashv\vdash h\}, h$ | opinion: pro |
| 2. con: $\{e_2 \dashv\vdash b, b \dashv\vdash \text{not-}h\}, \text{not-}h$ | opinion: none |
| 3. pro: $\{e_1 \ \& \ e_2 \dashv\vdash \text{not-}b\}, \text{not-}b,$ | opinion: pro |

establishing h . (See figure 4.)

III.1.1. Preliminaries for c-, s-, and d-rationales.

In procedural models, citing a rationale is a prelude to an attack. It is a necessary part of what makes the subsequent argument relevant and sufficient. It alters the context so that the subsequent

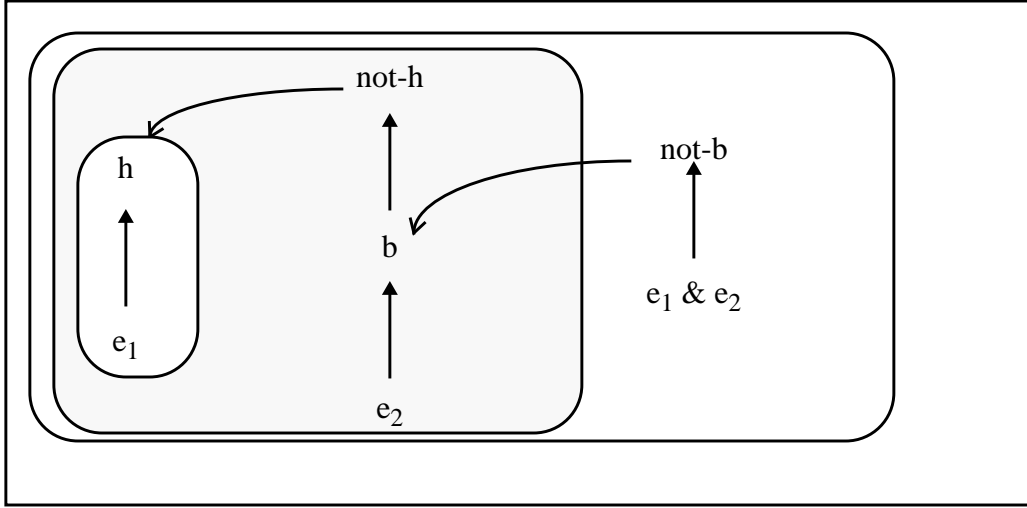


Fig. 4: Three arguments introduced one at a time according to protocol.

argument is a legitimate move by the player. A rationale, cited, permits a_{i+1} to respond to a_i . Without citing the rationale, a_{i+1} is not a legitimate successor of a_i .

In declarative models, there are at least two ways to regard rationales.

Citing rationales may alter L or C , restating the formulation of rules and cases to which a player is willing to agree. In this way, the appropriate basis for warrant is $\text{Args}(L', C', E)$ instead of $\text{Args}(L, C, E)$.

The second way is to suppose that the rationale, cited, changes the relations of defeat among the arguments, def , the binary relation on $\text{Args}(L, C, E)$ that is defined by the underlying system of argument. Contemporary systems regard argument a_1 as defeating argument a_2 on syntactic criteria, or according to an externally supplied ordering, $\text{extord}(\text{Args}(L, C, E))$. The latter might arise from orderings on its constituents: $\text{extord}(L)$, $\text{extord}(C)$, $\text{extord}(E)$. With rationales, the set of cited rationales, R , becomes a new index in the determination of defeat. The derivation of the defeat relation:

$$\text{extord}(L), \text{extord}(C), \text{extord}(E) \rightarrow \text{extord}(\text{Args}(L, C, E)) \rightarrow \text{def};$$

becomes:

$$\text{extord}(L), \text{extord}(C), \text{extord}(E), R \rightarrow \text{extord}(\text{Args}(L, C, E)) \rightarrow \text{def}.$$

If R is static, then R is only important to those who study these mappings. This is why rationales do not have much importance in declarative settings. Taking R to be dynamic, as a sequence R_i , one may as well return to a procedural model. We will just assume a procedural model of argument.

Defn. Citable Rationales, $R(l)$, $R(c)$. Let $R(l)$ and $R(c)$ respectively be all citable rationales for a defeasible rule, l in L , and a precedent-setting case, c in C .

The form of a rationale depends on the kind of rationale, as elaborated below. Practically, the most

difficult part of arguing with rationales will be discovering and formulating the set $R(l)$, for a rule l , or $R(c)$, for a case, c . Even when an authoritative decision is explained, and even when legislative deliberation is well preserved, formal expression of rationales as R is problematic. The assumption here is the usual assumption among formalists: just as evidence, E , wording of rules, L , and documentation of cases, C , can be expressed in formal language, so too can rationales, R . To address the problem more fully would be more ambitious than what is usually attempted by formalists.

The intention is that $R(l)$ will be a singleton set for most rules (likewise for $R(c)$). There could be multiple citable rationales, and the difference could matter to the course of a particular dispute. There could also be no citable rationale.

If $R(c)$ or $R(l)$ is not a singleton for the relevant case or rule, then the parties can dispute the appropriate rationale. Since we suppose no information that would allow adjudication of such a dispute, the simplest reasonable rule will be adopted.

If a rationale is raised for a rule or case in an argument, and there is a multiplicity of possible rationales, then it suffices that the player who makes use of the original argument can make progress under one of the many rationales. For example, suppose pro has the burden to establish h . Pro produces an argument A for h , using rule l . If l has multiple rationales in $R(l)$, let them be named $rat_1(l)$, $rat_2(l)$, etc.; if they are of different types, name them $c-rat_1(l)$, $d-rat_2(l)$, etc. In the suggested protocol it suffices for pro if h can be established after unpacking pro's choice of the permitted rationales. Call this the *User's Prerogative Assumption*.

The procedural rules that implement this assumption merely complicate the presentation in an uninteresting way. The simplest way of making the assumption of user's prerogative is to assume that all $R(c)$ and $R(l)$ are either singletons or empty sets, for every rule l or case c .

Assumption. Rationales Unique/User's Prerogative. For each l , if $R(l)$ is non-empty, then it is a singleton. Likewise for each c and $R(c)$.

III.1.2. Structures for c-, s-, and d-rationales.

For rules of the form "p is defeasible reason for q", rationales take the following forms.

A c-rationale is typically an argument, with premises p and conclusion q . There may be a background, B , against which this argument was made. The intention in adopting the rule is that usually B will be present in contexts in which the rule is applied. A c-rationale may thus be an argument from p and B to q . Such an argument is as above, a 2-tuple:

Defn. Compressions of Rules, c-rat. $c-rat(p \dashv\vdash q) = \langle T, q \rangle$,
 where T is a subset of L ;
 $c-rat(p \dashv\vdash q)$ is in $Args(T, \{\}, \{p\} \cup B)$.
 (See figure 5.)

An s-rationale is formally identical to a c-rationale except that the argument takes a particular form. In a c-rationale, several rules are used in the argument, which is compressed. T is not a singleton

set. An s-rationale uses a single guiding rule, a principle. It uses supplementary rules in L that permit the implementation of the principle, and probably relies on some sentences in B.

Defn. Specializations of Rules, s-rat. $s\text{-rat}(p \dashv\vdash q) = \langle \{\text{princ}, \text{impl}\}, q \rangle$,
 where princ and impl are in L;
 $c\text{-rat}(p \dashv\vdash q)$ is in $\text{Args}(T, \{\}, \{p\} \cup B)$.
 (See figure 6.)

A d-rationale of a case c is a set of arguments, $d\text{-rat}(c)$, that has a certain structure. These arguments are typically not from $\text{Args}(L, C, E)$ but are from a different set, $\text{Args}(L', C', E')$. The applicability of the case to the current situation depends crucially on whether the arguments in $d\text{-rat}(c)$ are in both of these sets, not just in $\text{Args}(L', C', E')$. For a d-rationale, $d\text{-rat}(c)$ is a set of arguments that warrants the decision of the case.

Defn. Disputations from Past Cases, d-rat. $d\text{-rat}(c) = \{ \text{arg}_1, \dots, \text{arg}_k \}$,
 where each arg_i is in the intersection of $\text{Args}(L, C, E)$ and $\text{Args}(L', C', E')$,
 where $\langle L', C', E' \rangle$ describes the context of the prior case.
 (See figure 7.)

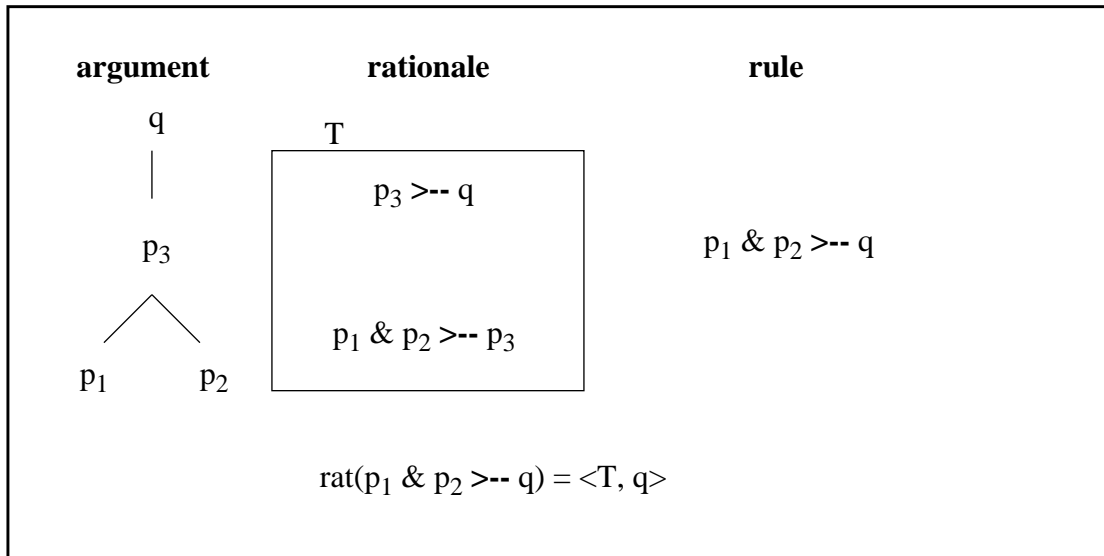


Fig. 5: A (c)ompression rationale for a rule is an argument compressed.

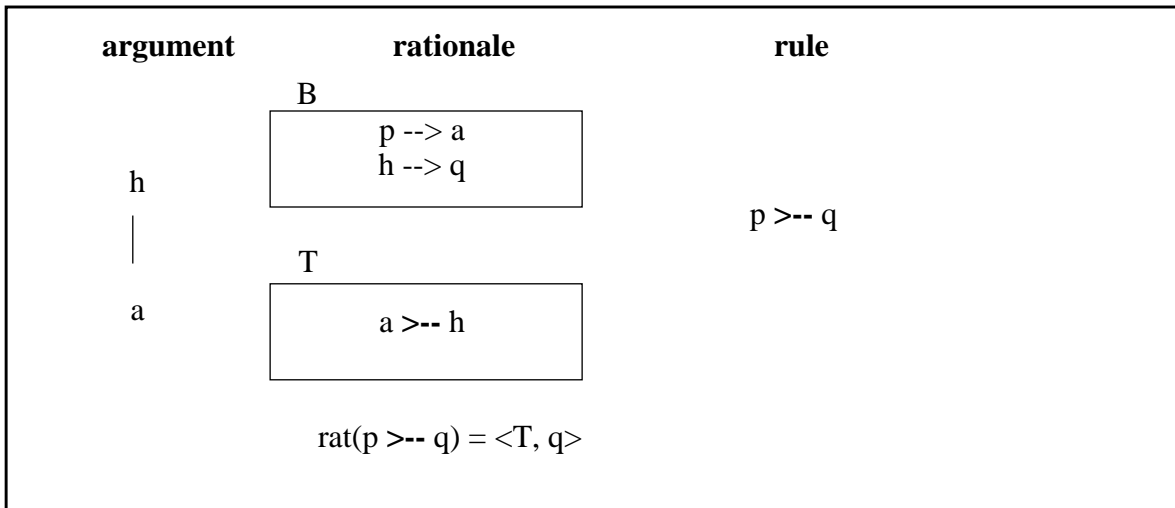


Fig. 6: A (s)pecialization rationale for a rule is a principle used particularly.

III.1.3. Attacks for c-, s-, and d-rationales.

Let ArgRec_n be the recognized arguments at a stage n in the sequence of moves.

In a simple disputation where moves do not refer to rationales and all arguments are well-formed members of $\text{Args}(L, C, E)$, ArgRec_n is simply the union of all arguments introduced in all moves, m_1, \dots, m_n . In disputations where well-formedness of an argument can be called into question (which includes disputations that allow rationale-based attacks on arguments), ArgRec_n expands and contracts as the disputation proceeds.

ArgRec_n includes both defeated and undefeated arguments. It is not the “arguments in force” concept that Vreeswijk defines [1993], which is useful in analyzing warranted conclusions. It is preliminary to determining which arguments are in force. An argument that was introduced in the dialogue but has subsequently been excluded from ArgRec cannot even be considered for its

properties of defeat relative to the other arguments.

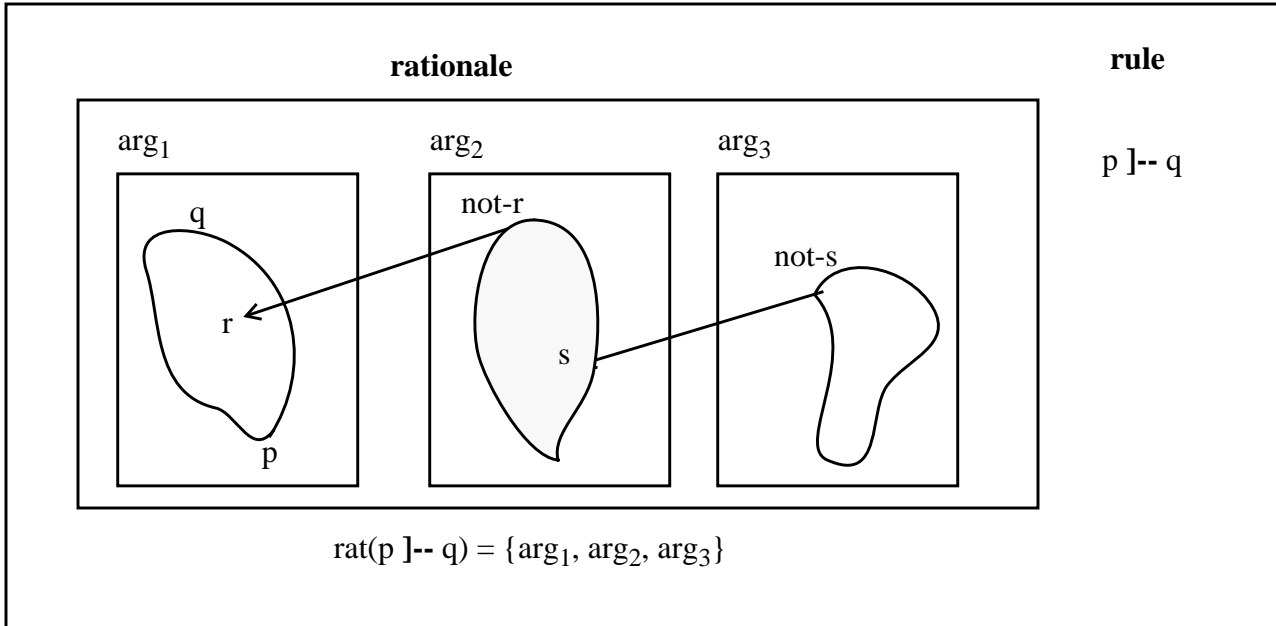


Fig. 7: A (d)isputation rationale for a rule from a case is a record of disputation.

Here, r is an intermediate conclusion in arg_1 , attacked by arg_2 . arg_1 is reinstated by arg_3 's attack on arg_2 at s . The rule includes only the assumptions from arg_1 . It could have included the assumptions in arg_2 and arg_3 as well.

Defn. Additions to the Argument Record, ArgRec: Part I: Basic Moves. ArgRec_k is defined inductively:

- i. an argument in m_0 is in ArgRec_0 ;
- ii. if m_i does not contain a statement of rationale, then for an argument a_i in m_i , ArgRec_i contains $\text{ArgRec}_{(i-1)} \cup a_i$;
- iii. if m_i contains a statement of a rationale, then this is either a rationale-based attack on prior argument, or else a reinstatement of an argument that has suffered a rationale-based attack.

A statement of a rationale in a move requires the naming of the argument that is attacked or defended. It also requires the naming of the rule or case, the rationale of which is at issue.

Defn. Rationale Moves, @. Let $@_i$ indicate that i is a move that uses a rationale. For a move i such that $@_i$, let a_i be the argument concerned, and let l_i or c_i be the rule or case, as appropriate, in question. Let r_i be the rationale, whether it is a c-, s-, or d-rationale. Let player_i be the player who moved in move i .

Trivial well-formedness requires that the attacked argument has occurred, that the rule occurs in the argument, and that the rationale is recognized:

Additional Constraints on Rationale Moves.

If $@_i$, then

- iii.a. a_i must be in ArgRec_k for some $k < i$ (k need not be $i-1$ since the defense against rationale-based attacks must be possible);
- iii.b. l_i or c_i must be a part of a_i ;
- iii.c. r_i must be in $R(l_i)$, or $R(c_i)$.

Note that r_i and a_i may have occurred together in a previous move, $j < i$, where both $@_i$ and $@_j$. This seems to permit repetition of moves, but m_i still might not be the same as m_j . Repetition will be prevented by a requirement that moves be effective.

The same rule or case can repeatedly be the target of rationale-based attacks if it occurs in distinct arguments. This is simpler than calling all arguments based on a rule or case into question at once. Note that both players might be relying on the same argument, a_i .

Apparently, the immediate effect of the attack is to remove a_i from ArgRec_i . That is, apparently, $\text{ArgRec}_i = \text{ArgRec}_{(i-1)} - \{a_i\}$. But the move might modify the form of the argument removing a_i and substituting a revised argument in its stead. So apparently,

Defn. Changes to The Argument Record, ArgRec: Part II: Effects of Rationale Moves.

- iv. $\text{ArgRec}_i = \text{ArgRec}_{(i-1)} - \{a_i\} \cup \text{argrevs}$, where argrevs contains a modified form of a_i , or else is the empty set;

For a c-rationale or s-rationale, which is an argument $\langle T, h \rangle$, argrevs contains just the argument that uses $\langle T, h \rangle$ as a subargument instead of the rule, l_i , that is attacked. So if $\langle T_0, h_0 \rangle = a_i$, then argrevs contains the argument in which a subargument has been substituted for the compressed rule: $\langle T_0 \cup T - \{l_i\}, h_0 \rangle$. This substitution could be denoted: $\text{subst}(a_i, l_i, T)$. If indeed it is an argument (satisfies minimality and consistency conditions), it is the sole element of argrevs . Otherwise, the statement of the rationale destroys a_i and there is no revised argument added in its place; argrevs is the empty set.

For a d-rationale, argrevs is r_i , a set of arguments. m_i must contain a new argument, attack_i , which attacks argrevs and changes the current opinion, dec_i . In a d-rationale, the decision of the case, $\text{dec}(c_i)$, is warranted among the arguments recorded for the case, r_i . That is, enough arguments were recorded to support the decision outright. So m_i must be such that $r_i \cup \{\text{attack}_i\}$, taken as a set of arguments, would not have warranted the old decision outright, $\text{dec}(c_i)$.

Defn. The Revised Argument(s), argrevs, for c-, s-, and d-rationales.

- iv.a. If $@_i$ cites a c-rationale or s-rationale, then $\text{argrevs} = \{ \text{subst}(a_i, l_i, T) \}$ if $\text{subst}(a_i, l_i, T)$ is a well-formed argument, otherwise, $\text{argrevs} = \{ \}$;

- iv.b. If $@_i$ cites a d-rationale, then $\text{argrevs} = \text{d-rat}(c_i) \cup \{\text{attack}_i\}$, where c_i is the case attacked, and attack_i is an argument that alters the status of a_i 's conclusion.

This completes the inductive definition of ArgRec_i .

Finally, having defined ArgRec_i in terms of m_i and $\text{ArgRec}_{(i-1)}$, we require the move m_i to be effective: it must alter current opinion so that player_i 's position is improved. When argrevs is part of m_i , m_i must sometimes contain a response to the revised argument as well.

Responses to these simple kinds of rationale-based attacks are just moves that augment ArgRec .

For example, for a d-rationale, all of the arguments recorded of the past case have been entered into ArgRec . Among them was an argument, arg_w , that warranted the decision of the case. Also included is the opponent's argrevs , which has somehow changed the status of arg_w . To respond substantively, simply give an argument that restores arg_w 's ability to support its conclusion again. Or simply retreat from the attacked precedent-based argument and provide a new, unrelated argument to support the old decision.

III.1.4. Simple Symbolic Examples for c-, s-, and d-rationales.

Example. A dialogue without a rationale.

- | | |
|--|-----------------------------------|
| 1. pro: $\text{arg}_1 = \langle \{ b \rangle \text{--} a \}$, | i.e., b is reason for a; |
| $b! \therefore a \rangle$ | i.e., b is asserted; therefore a. |

$\text{ArgRec}_1 = \{ \text{arg}_1 \}$; $\text{dec}_1 = \text{pro}$.

- | | |
|--|---|
| 2. con: $\text{arg}_2 = \langle \{ c \rangle \text{--} d,$ | i.e., c is reason for d; |
| $d \ \& \ b \rangle \text{--} \text{not-a} \}$, | i.e., d and b are joint reason for not-a; |
| $c! \ b! \therefore \text{not-a} \rangle$ | i.e., c and b; therefore not-a. |

$\text{ArgRec}_2 = \{ \text{arg}_1, \text{arg}_2 \}$; $\text{dec}_2 = \text{none}$.

- | | |
|--|---|
| 3. pro: $\text{arg}_3 = \langle \{ c \ \& \ e \rangle \text{--} \text{not-d} \}$, | i.e., c and e are joint reason for not-d; |
| $c! \ e! \therefore \text{not-d} \rangle$ | i.e., c and e; therefore not-d. |

$\text{ArgRec}_3 = \{ \text{arg}_1, \text{arg}_2, \text{arg}_3 \}$; $\text{dec}_3 = \text{pro}$.

Here, arguments are 2-tuples (a set of rules paired with a conclusion). We have embellished the second argument. Instead of just giving the conclusion of the argument, we give the evidence on which the argument rests (marked with an exclamation point) and write “ \therefore ” to separate the conclusion. Rules are given in their sentential form, $a \text{--} b$ instead of using the pair $\langle a, b \rangle$, and quotes are suppressed. Note that we use “=” to give a name to a structure. This is not to be confused with an assertion.

At move 4, it is con's turn to move and opinion favors pro: arg_3 defeats arg_2 , thus reinstating arg_1 . If con cannot move at this point, pro wins.

Example. A c-rationale.

1. pro: $\text{arg}_1 = \langle \{ b \dashrightarrow a \},$
 $b! \therefore a \rangle$ “a because of b.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con: “your argument is really:”
- 2.1. $\text{c-rat}_1(b \dashrightarrow a) = \langle \{ b \dashrightarrow c,$
 $c \dashrightarrow a \},;$
 $b! \therefore a \rangle$ “c because of b, a because of c.”
- 2.2. $\text{Attack} \langle b \dashrightarrow a, \text{arg}_1 \rangle$ “because that is the rationale of the rule in your argument, and now here is my counterargument:”
- 2.3. $\text{arg}_2 = \langle \{ b \& d \dashrightarrow \text{not-c} \},$
 $b! d! \therefore \text{not-c} \rangle$ “not-c because of b and d.”

Con states a rationale for a rule, which presumably is the legitimate member of $R(b \dashrightarrow a)$. Con must state the argument being attacked. The effect is to replace the compressed argument with the uncompressed, but this is still not a sufficient response for con. So con provides arg_2 , an attack on the uncompressed argument, the *raison d’etre* for stating the rule’s rationale. @₂. Note $\text{pro} = \text{user}(b \dashrightarrow a, \text{arg}_1)$. $\text{con} = \text{attacker}(b \dashrightarrow a, \text{arg}_1)$.

$\text{ArgRec}_2 = \{ \text{arg}_1 \text{rev}, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

where

$\text{arg}_1 \text{rev} = \text{subst}(\text{arg}_1, b \dashrightarrow a, \text{c-rat}_1(b \dashrightarrow a))$

3. pro: $\text{arg}_3 = \langle \{ b \& d \& e \dashrightarrow c,$
 $c \dashrightarrow a \},$
 $b! d! e! \therefore a \rangle$ “ok, here is a new argument for c.”
“c because of b, d, and e; a because of c.”

Pro provides a new argument for a, arg_3 , that is undefeated in ArgRec_3 . The new argument for c is specific enough to defeat arg_2 , which would reinstate arg_1 , but arg_1 has been flushed from the ArgRec in favor of $\text{arg}_1 \text{rev}$.

$\text{ArgRec}_3 = \text{ArgRec}_2 \cup \{ \text{arg}_3 \}; \text{dec}_3 = \text{pro}.$

Example. An s-rationale.

1. pro: $\text{arg}_1 = \langle \{ b \dashrightarrow a \},$
 $b! \therefore a \rangle$ “a because of b.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con: “your argument really involves d and c: a achieves e; b is an instance of d.”

2.1. $s\text{-rat}_1(b \succ\text{-} a) = \langle \{ d \succ\text{-} e \}, b! (d \text{ or not-}b)! (a \text{ or not-}e)! \therefore a \rangle$
 “this is the real form of your argument.”

2.2. $\text{Attack} \langle b \succ\text{-} a, \text{arg}_1 \rangle$

2.3. $\text{arg}_2 = \langle \{ d \& f \succ\text{-} \text{not-}e \}, d! f! \therefore \text{not-}c \rangle$, “in light of this, my argument for not-e is a counterargument.”

$\text{ArgRec}_2 = \{ \text{arg}_1\text{rev}, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

where

$\text{arg}_1\text{rev} = \text{subst}(\text{arg}_1, b \succ\text{-} a, s\text{-rat}_1(b \succ\text{-} a))$

Here, arg_2 defeats arg_1rev because it is more specific. But arg_2 does not defeat arg_1 on specificity.

3. pro: $\text{arg}_3 = \langle \{ d \& f \& g \succ\text{-} e \}, b! (d \text{ or not-}b)! f! g! (a \text{ or not-}e)! \therefore a \rangle$
 “ok, here is a separate argument for a, using a more specific justification of e.”

$\text{ArgRec}_3 = \text{ArgRec}_2 \cup \{ \text{arg}_3 \}; \text{dec}_3 = \text{pro}.$

Notice how similar this example is to the example of a c-rationale and recovery from a c-rationale-based attack.

Example. A d-rationale.

1. pro: $\text{arg}_1 = \langle \{ b \& c \text{]--} a \}, b! c! \therefore a \rangle$, “in virtue of b and c, and precedent, my claim is a.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

Here,]-- is used instead of >-- to indicate that this rule is extracted from a case.

2. con:

2.1. $d\text{-rat}_1(b \& c \text{]--} a) = \{ da_1, da_2, da_3 \}$
 “here are the main arguments from case being claimed as precedent.”

where

$da_1 = \langle \{ b \succ\text{-} a \},$ “the main argument was for a.”

determines a few things: whether meta-arguments are undefeated and supporting, hence, whether their meta-claims warranted; whether the arguments that make use of the meta-claims are undefeated and supporting, hence, whether their object-level claims are warranted. This assumption is made for two reasons: (a) to define the envisioned procedural system would be a separate study not crucial to the picture of rationales that is drawn here; and (b) the use of meta-argument here provides requirements for those who would attempt to define such a system.

III.2.1. Preliminaries for r- and f-rationales.

For an r-rationale, the crucial assertion is an assertion about the relation among arguments, a meta-assertion. Arguments that were undefeated in the cited case should continue to be undefeated in the current case in order for the past decision to have the proper influence.

For an f-rationale, two kinds of meta-assertion are crucial. They are that a case should have a certain decision, and that a rule fits a set of cases.

There are a few kinds of attacks on rules with f-rationales that we can model and a few that we cannot model. Even for the few that we do model, we must introduce negated assertions of a sentence being a reason for another sentence, e.g. $\text{not}(b \succ\!\!\succ a)$.

An r-rationale is similar to a d-rationale. It is at least a set of recorded arguments from the prior dispute. For an r-rationale of a case c , $\text{r-rat}(c)$ is a set of undefeated arguments that interfere with each other. It is this interfering set of arguments that the decision of the case resolves. $\text{r-rat}(c)$ also contains the implicit argument that when those arguments are undefeated, a certain decision is appropriate. An assumption here is that there is no dispute that the arguments are undefeated (or what is nearly the same, that their status of being undefeated can be determined easily). If the defeat relations among arguments are clear, then this assumption is reasonable.

An argument so far has been a set of defeasible rules from L , and a conclusion derivable from those rules using the accepted (incontrovertible) evidential claims. Henceforth, the set of rules can contain rules from L and meta-rules. These meta-rules involve a specific kind of meta-claim: that a certain set of arguments contains only arguments that are undefeated, or not. They relate this meta-claim to a different kind of meta-claim: a defeasible rule. For example, instead of allowing just

$$a \succ\!\!\succ b$$

we allow as well

$$\text{undefeated-members}(S) \succ\!\!\succ (a \succ\!\!\succ b).$$

The first occurrence of “ $\succ\!\!\succ$ ” above is a relation in the meta-meta-language, and the second occurrence of “ $\succ\!\!\succ$ ” is the relation in the meta-language used throughout the preceding sections. Derivations are now allowed to make use of an argument’s rules and meta-rules (arguments may use both kinds of “ $\succ\!\!\succ$ ”). The mathematical demands on the theory of argument imposed here are serious and not to be ignored. Fortunately, implementations have shown that progress can be made with systems defined in this way, e.g., [Sartor, 1993b], despite the mathematical and philosophical

conundrums.

Defn. A Set of Undefeated Arguments, undef. $undef(S)$ asserts of a set of arguments, $S = \{A_1, A_2, \dots, A_n\}$ that each of those arguments contained in S can be made in the current setting and is undefeated.

If S is such that $undef(S)$, then S must be contained in $Args_i(L, C, E)$. The predicate, $undef$, is thus dependent on all of the indexes: i , L , C , and E . Since i is the most important index here, write $undef_i(S)$ when it is important to distinguish the stage at which $undef$ is claimed. (See figure 8.)

This is not the final formulation for r -rationales. As Ashley's HYPO (1989) makes clear, a precedent can be used in the presence of additional arguments *pro*. If a case decides that argument A_1 for h outweighs argument A_2 for *not-h*, then if A_1 , A_2 , and an additional argument for h , A_3 , can be made in a new setting, the decision of the prior case is still relevant. The precedent can also be used in a context where there are fewer arguments *con* than those that figured in the past resolution. Hence, it need not be required that exactly those undefeated arguments in the past case be undefeated in the present case.

Instead, of the predicate $undef_i(S)$, suppose there are two predicates: $minpro-undef_i(S_1)$ and $maxcon-undef_i(S_2)$. The arguments in each of the sets S_1 and S_2 were weighed in the prior decision, and arguments in S_1 taken together were more persuasive than those in S_2 . If at stage i , at least the arguments in S_1 are undefeated, and if no more than the arguments in S_2 for *con* are undefeated, then there is defeasible reason for using the case (or more precisely, the rule extracted from the case). If the arguments in S_1 cannot be maintained as undefeated at stage i , then the precedent cannot be used. If there are additional arguments *con*, which were not weighed previously, then the case is again inapplicable.

Defn. Current Disposition of Past Arguments, minpro-undef, maxcon-undef. If S is a set of arguments such that $undef(S)$, then $minpro-undef(S_1)$ and $maxcon-undef(S_2)$ if S_1 and S_2 are disjoint subsets of S , and S_1 contains the arguments *pro*, S_2 contains the arguments *con*.

A formal account of determining when an argument should be considered *pro* or *con* is owed. As in HYPO, this determination is trivial if the decision of the case is h and all arguments in S_1 and S_2 are for h and *not-h*, respectively. Determining whether an argument is *pro* or *con* can be inductively defined in an obvious way, and will not be done here. There will be arguments that support both *pro* and *con* arguments, and these most naturally contribute to $minpro-undef_i$ and are excluded from $maxcon-undef_i$.

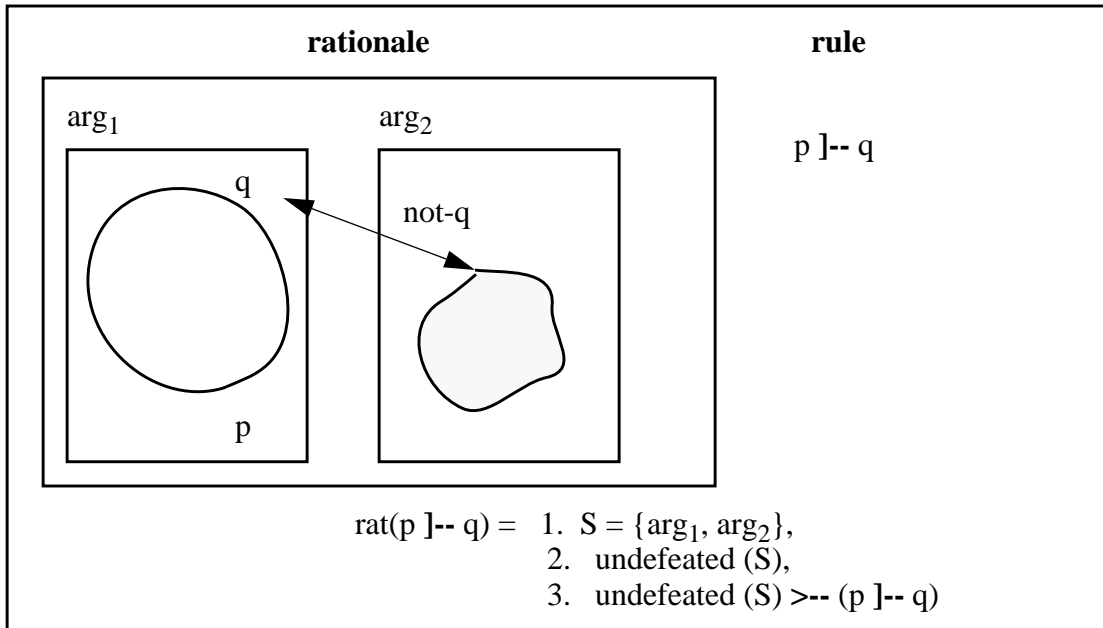


Fig. 8: A (r)esolution rationale is like a (d)isputation rationale but includes a statement that the arguments continue to stand in their past relation, and a statement that arguments so standing justify the rule. This is not the final formulation of r-rationales.

An f-rationale is a set of cases, not all of which might be from C. Let $HC \cup C$ be the actual cases augmented by hypothetical cases. HC are cases that did not actually occur, but are cases upon which there is agreement of the hypothetical decisions given the hypothetical facts. Assume that the assertion that a case and its decision are agreeable to all parties is incontrovertible. Write $ok(\text{case}_1)$ if case_1 is so agreeable. Once again, we do not seek to model every kind of dispute. Here, we choose not to model disputes over whether a case should have a certain outcome. Some of these disputes could be modeled easily, some would require a complicated recursion, and some are not obviously subject to reason at all.

Instead of introducing a language in which cases are described, suppose simply that there is a relation $\text{fit-s}(l, c)$, which holds when a defeasible rule fits a case c . For example, the rule $l_1 = \text{"if } f_1 \text{ and } f_2, \text{ then } h\text{"}$ fits those cases wherein $f_1, f_2,$ and h hold. It also fits those cases wherein h does not hold, but at least one of f_1 or f_2 also does not hold. The rule does not fit a case wherein $f_1, f_2,$ and $\text{not-}h$ are all true. This is fit *simpliciter*. A more robust concept of a defeasible rule's fit to a case takes account of context.

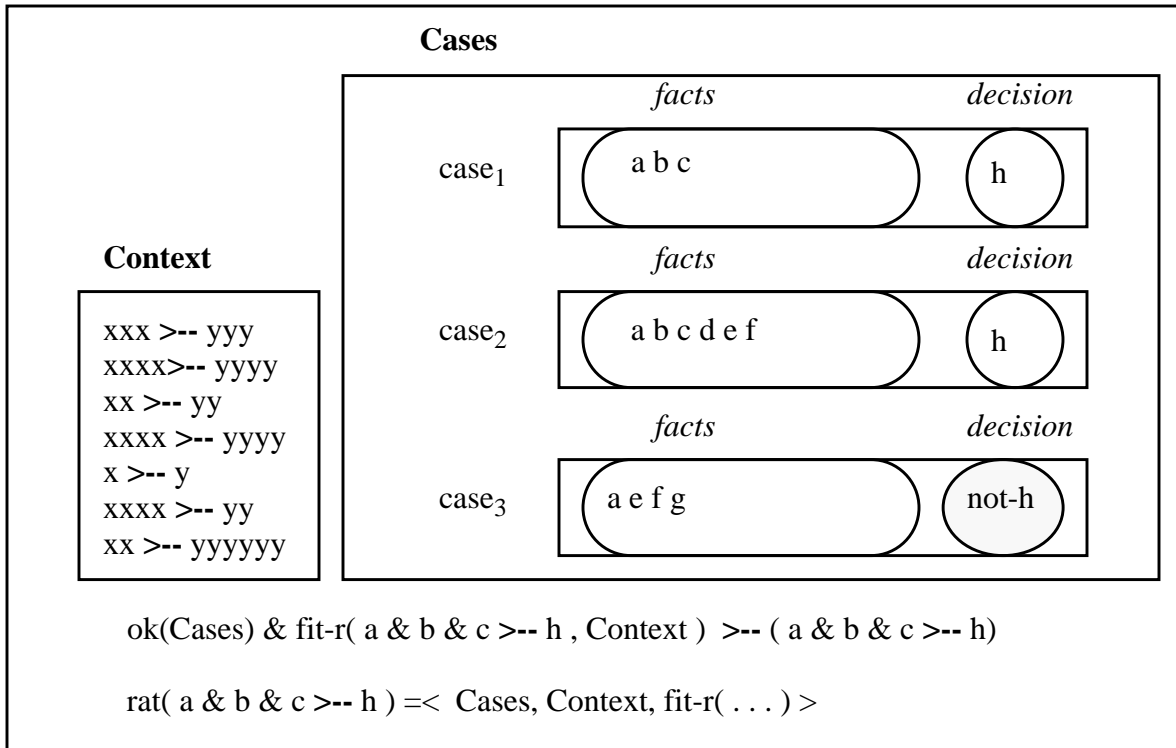


Fig. 9: A rationale of a rule's (f)it is a set of cases, a context of rules, and a statement that there is fit of rule to cases given the context. that

In the context of a set of other defeasible rules, LCont, l might fit a case c even if it does not fit *simpliciter*. This is because LCont may provide through more specific rules an explanation of why c does not conform to l, why it is not the case that fit-s(l, c). So a rule may fit, *robustus*, fit-r(l, c, LCont) when fit-s(l, c) or when there is a more specific rule, l', in LCont which does fit c. The whole apparatus of arguments which permits chaining of rules could be applied here to explain why l does not fit c, *simpliciter*, but is allowed not to fit by a superior chain of rules in LCont. Under the criteria of syntactic superiority advanced in [Simari & Loui, 1992], [Prakken, 1993], and [Loui et al., 1993a], no chain of rules in LCont will be superior to l unless a single rule in LCont is superior to l. But under other systems, a recursion of disputation is possible here, and woe be to the formalizer who seeks to model this recursion. (See figure 9.)

Defn. A Rule Fits a Case or Cases, fit-s, fit-r. For a rule l, a case c, and a set of rules, LCont:

fit-s(l, c) holds when l fits c.

fit-r(l, c, LCont) holds (i) when fit-s(l, c) or (ii) when there is a rule, l', such that l' is more specific than l, l' in LCont, and l' fits c.

We also write fit-s(l, C), for a set of cases, C, if fit-s(l, c) for every case in the set of cases, C. Similarly for fit-r(l, C).

Arguments of fit based on a larger number of cases are considered better arguments. Likewise, arguments of fit based on a larger context are better arguments. Fit is assumed incontrovertible.

III.2.2. Structures for r- and f-rationales.

Formally, an r-rationale is a set of arguments that occurred in the case, a true assertion that all of those arguments are undefeated with respect to one another (none is defeated), and a meta-reason that connects the undefeatedness of certain arguments to the rule putatively extracted from the case. The arguments are partitioned into minpro and maxcon, as discussed above. Technically all that are required are the two sets of arguments, S_1 and S_2 , that occurred in the resolution of the case. The rest can be recovered from context. Making the other parts explicit adds clarity.

Defn. Resolution of Arguments, r-rat. $r\text{-rat}(c) = \langle$
 1. $S = S_1 \cup S_2$,
 2. $\text{minpro-undef}(S_1), \text{maxcon-undef}(S_2)$,
 3. $\text{minpro-undef}(S_1) \ \& \ \text{maxcon-undef}(S_2) \ \>-- \ 1$
 \rangle ,
 where S is a set of arguments partitioned into arguments pro, S_1 , and con S_2 .

The claim of undefeatedness must be true, and the arguments in S must be properly relevant to c . To define proper relevance, consider the rule extracted from the case to be: $l = l_1 \ \>-- \ l_2$. There must be an argument in S for l_2 . We do not insist that all arguments in S be possible with just l_1 as evidence. Nor must there be some argument in S that requires all of l_1 to be made. Much more weakly, l_1 must be relevant to S ; it should contain no superfluous literals.

An f-rationale is a set of cases together with a claim that the rule fits (*robustus*) that set of cases with respect to a context. $\langle HC \cup C, LCont \rangle$ is the f-rationale for a rule l , if and only if for every c in $HC \cup C$, $\text{fit-r}(l, c, LCont)$. $LCont$ must be a subset of L .

Defn. The Cases the Rule Fits, f-rat. $f\text{-rat}(p \ \>-- \ q) = \langle$
 1. C ,
 2. Context,
 3. $\text{fit-r}(p \ \>-- \ q, \text{Cases}, \text{Context})$
 \rangle ,
 where C is a set of cases, and Context is a set of rules that includes $p \ \>-- \ q$.

III.2.3. Attacks for r- and f-rationales.

To attack an argument using a rule with r-rationale, cite the rationale and name the argument and rule attacked. Then substitute a revised argument into ArgRec_i that makes explicit the connection between the undefeated arguments resolved by the case and the rule extracted therefrom. The modified argument is just like the earlier argument except that a meta-rule has been put in place of the object-level rule. The meta-rule makes explicit the origin of the object-level rule. This allows subsequent attack of the statement that S contains undefeated arguments.

To attack the statement $\text{minpro-undef}(S)$, provide a new argument or set of arguments, S' , that are

arguments that can be made in the current case. Then claim $\text{not}(\text{minpro-undef}(S))$ as a result of the presumed system of determining defeat among arguments. If S' contains an argument that defeats an argument in S , for example, then not every member of S will be undefeated (some member will now be defeated). Similarly, to attack $\text{maxcon-undef}(S)$, provide a new argument or set of arguments, S' that can be made in the current case. Then claim $\text{not}(\text{maxcon-undef}(S))$ because S' contains a new undefeated argument con.

To attack an argument using an f-rationale, state the f-rationale, then alter either the cases or the context with respect to which fit is claimed. We have assumed that claims of fit are incontrovertible, since we are providing no theory of how this is determined. Adding a case that the rule does not fit is the usual attack.

III.2.4. Simple Symbolic Examples for r- and f-rationales.

Example. An r-rationale with maxcon-undef attacked.

$$1. \text{ pro: } \arg_1 = \langle \{ b \ \& \ c \} \text{--} a \rangle, \\ b! \ c! \ \therefore \ a \rangle$$

“in virtue of b and c, and precedent, my claim is a.”

$$\text{ArgRec}_1 = \{ \arg_1 \}; \text{dec}_1 = \text{pro.}$$

$$2. \text{ con:}$$

“that prior decision weighed two arguments, where the argument based on b, mediated by d was judged better than the argument based on c.”

$$2.1. \text{ r-rat}_1(b \ \& \ c \]\text{--} a) = \\ \langle \{ da_1, da_2 \}, \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \}) , \\ \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \}) \text{>--} (b \ \& \ c \]\text{-} a) \rangle$$

where

$$da_1 = \langle \{ b \text{>--} d \\ d \text{>--} a \} , \\ b! \ \therefore \ a \rangle,$$

$$da_2 = \langle \{ c \text{>--} \text{not-}a \} , \\ c! \ \therefore \ \text{not-}a \rangle$$

$$2.2. \text{ Attack } \langle b \ \& \ c \]\text{--} a, \arg_1 \rangle$$

$$2.3. \arg_1 \text{rev} = \langle \\ \{ \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \}) \text{>--} (b \ \& \ c \]\text{-} a) \} , \\ b! \ c! \ \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \})! \ \therefore \ a \rangle$$

“in order to defend the decision,

you must at least be able to argue for a, and the opposition must have no better case than before.”

2.4. $\text{arg}_2 = \langle \{ e \text{>--} f, \\ f \text{>-- not-a } \}, \\ e! \therefore \text{not-a } \rangle$

“but here is a new argument that adds to con’s case in a non-trivial way and requires a re-weighing of arguments.”

2.5. $\text{not}(\text{maxcon-undef}(\{da_2\}))$

$\text{ArgRec}_2 = \{ \text{arg}_1\text{rev}, da_1, da_2, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

Note that among da_1 and da_2 , neither is defeating. The opinion in this case is significant because it creates a preference that cannot be found in $\text{extord}(\text{Args}(L, \{ \}, E))$ or in defeat, that is, among the syntactically determinable or externally supplied orderings among arguments. The adjudicator chose between competing arguments that could not be decided purely on form. Con revises pro’s argument arg_1 by producing arg_1rev which exposes the meta-reasoning. Then arg_2 is produced, which is an additional argument con, and is undefeated. So 2.5 is assertible, $\text{not}(\text{maxcon-undef}(\{da_2\}))$, which is one of the requirements for using the precedent.

Example. An r-rationale with minpro-undef attacked.

1. pro: $\text{arg}_1 = \langle \{ b \ \& \ c \]\text{--} a \}, \\ b! \ c! \ \therefore \ a \rangle$

“In virtue of b and c, and precedent, my claim is a.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con:

“that prior decision weighed two arguments, where the argument based on b, mediated by g was judged better than the argument based on c, mediated by d.”

2.1. $\text{r-rat}_1(b \ \& \ c \]\text{--} a) = \\ \langle \{ da_1, da_2 \}, \text{minpro-undef}(\{da_1\}) \ \& \ \text{maxcon-undef}(\{da_2\}), \\ \text{minpro-undef}(\{da_1\}) \ \& \ \text{maxcon-undef}(\{da_2\}) \text{>--} (b \ \& \ c \]\text{--} a) \rangle$

where

$da_1 = \langle \{ b \text{>--} g, \\ g \text{>--} a \}, \\ b! \ \therefore \ a \rangle,$

$da_2 = \langle \{ c \text{>--} d, \\ d \text{>--} \text{not-a } \},$

c! ∴ not-a>

2.2. Attack < b & c]-- a, arg₁ >

2.3. arg₁rev = <

{ minpro-undef({da₁}) & maxcon-undef({da₂}) >-- (b & c]- a) },
b! c! minpro-undef({da₁}) & maxcon-undef({da₂})! ∴ a >

“in order to defend the decision,
you must at least be able to argue
for a, and the opposition must have
no better case than before.”

2.4. arg₂ = <{ b & e >-- not-g },
b! e! ∴ not-g >

“but here is a new argument”

2.5. not(minpro-undef({da₁}))

“which alters the status of the
argument which, at minimum, must
be possible for pro, in order to use
the results of the prior case.”

ArgRec₂ = { arg₁rev, da₁, da₂, arg₂ }; dec₂ = none.

Again, among da₁ and da₂, neither is defeating, so the opinion in this case is significant. Again, the adjudicator chose between competing arguments that could not be decided purely on form. Con revises pro’s argument arg₁ by producing arg₁rev which exposes the meta-reasoning as before. This time, arg₂ is produced, which defeats an essential part of the case, one of the minpro-undef arguments (in fact, the only minpro-undef argument), da₁. So 2.5 is assertible, not(minpro-undef({da₁})), which again is a requirements for using the precedent.

Example. An f-rationale attacked for fit and redeemed in larger context.

1. pro: arg₁ = <{ b >-- a },
b! ∴ a >

“a, because of b.”

ArgRec₁ = { arg₁ }; dec₁ = pro.

2. con:

2.1. f-rat₂(b >-- a) = < Cases₁, Context₁, fit-r(b >-- a , Cases₁, Context₁) >

“the rationale for the rule you use is
that it fits two main cases.”

where

Cases₁ = { case₁, case₂ }

case₁ = < { b, d }, a >

case₂ = < { not-b }, not-a >

Context₁ = { }

2.2. Attack $\langle b \text{ >-- } a, \text{ arg}_1 \rangle$

2.3. $\text{arg}_1\text{rev} = \langle$

$\{ \text{ok}(\text{Cases}_1) \ \& \ \text{fit-r}(b \text{ >-- } a, \text{Cases}_1, \text{Context}_1) \text{ >-- } (b \text{ >-- } a) \},$
 $\text{ok}(\text{Cases}_1)! \ \text{fit-r}(b \text{ >-- } a, \text{Cases}_1, \text{Context}_1)! \ b! \ \therefore \ a \rangle$

“your argument depends on those prior decisions being ok, and the fit of the rule to those cases.”

2.4. $\text{arg}_2 = \langle$

$\{ \text{ok}(\text{Cases}_2) \ \& \ \text{not-fit-r}(b \text{ >-- } a, \text{Cases}_2, \text{Context}_1) \text{ >-- } \text{not}(b \text{ >-- } a) \},$
 $\text{ok}(\text{Cases}_2)! \ \text{fit-r}(b \text{ >-- } a, \text{Cases}_2, \text{Context}_1)! \ \therefore \ \text{not}(b \text{ >-- } a) \rangle$

“but there is a third important case that the rule does not fit.”

where

$\text{Cases}_2 = \{ \text{case}_1, \text{case}_2, \text{case}_3 \}$

$\text{case}_3 = \langle b, c, \text{not-}a \rangle$

$\text{ArgRec}_2 = \{ \text{arg}_1\text{rev}, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

Con has added a case, case_3 , which the rule, $b \text{ >-- } a$, does not fit in Context_1 . Pro responds by exhibiting a larger context which accounts for the alleged deviation.

3. pro: $\text{arg}_3 = \langle$

$\{ \text{ok}(\text{Cases}_2) \ \& \ \text{fit-r}(b \text{ >-- } a, \text{Cases}_1, \text{Context}_2) \text{ >-- } (b \text{ >-- } a) \},$
 $\text{ok}(\text{Cases}_2)! \ \text{fit-r}(b \text{ >-- } a, \text{Cases}_2, \text{Context}_2)! \ b! \ \therefore \ a \rangle$

“perhaps the case is important, but it is subsumed under a different rule in the statutory context.”

where

$\text{Context}_2 = \{ b \ \& \ c \text{ >-- } \text{not-}a \}$

$\text{ArgRec}_3 = \{ \text{arg}_1\text{rev}, \text{arg}_2, \text{arg}_3 \}; \text{dec}_3 = \text{pro}.$

Example. Deepening an r-rationale-based attack.

1. pro: $\text{arg}_1 = \langle \{ b \ \& \ c \} \text{-- } a \},$
 $b! \ c! \ \therefore \ a \rangle$

“in virtue of b and c, and precedent, my claim is a.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con:

“that prior decision weighed two arguments, where the argument based on b, mediated by g was

judged better than the argument based on c, mediated by d.”

$$2.1. \text{r-rat}_1(b \& c]-- a) = \langle \{da_1, da_2\}, \text{minpro-undef}(\{da_1\}) \& \text{maxcon-undef}(\{da_2\}), \text{minpro-undef}(\{da_1\}) \& \text{maxcon-undef}(\{da_2\}) >-- (b \& c]- a) \rangle$$

where

$$da_1 = \langle \{ b >-- g, g >-- a \}, b! \therefore a \rangle,$$

$$da_2 = \langle \{ c >-- d, d >-- \text{not-a} \}, c! \therefore \text{not-a} \rangle$$

$$2.2. \text{Attack} \langle b \& c]-- a, \text{arg}_1 \rangle$$

$$2.3. \text{arg}_1\text{rev} = \langle \{ \text{minpro-undef}(\{da_1\}) \& \text{maxcon-undef}(\{da_2\}) >-- (b \& c]- a) \}, b! c! \text{minpro-undef}(\{da_1\}) \& \text{maxcon-undef}(\{da_2\})! \therefore a \rangle$$

“in order to defend the decision, you must at least be able to argue for a, and the opposition must have no better case than before.”

$$2.4. \text{arg}_2 = \langle \{ b \& e >-- \text{not-g} \}, b! e! \therefore \text{not-g} \rangle$$

“but here is a new argument”

$$2.5. \text{not}(\text{minpro-undef}(\{da_1\}))$$

“which alters the status of the argument which, at minimum, must be possible for pro, in order to use the results of the prior case.”

$$\text{ArgRec}_2 = \{ \text{arg}_1\text{rev}, da_1, da_2, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$$

This is as before: arg_2 is produced, which defeats an argument that must be minpro-undef. Now pro responds by attacking arg_2 and ejecting it from ArgRec_3 .

3. pro:

$$3.1. \text{arg}_3 = \langle \{ b \& e \& f >-- g \}, b! e! f! \therefore g \rangle$$

“ok, but your argument can be defeated”

$$3.2. \text{minpro-undef}(\{da_1\})$$

“which restores the minimum argument pro”

$\text{ArgRec}_3 = \{ \text{arg}_1\text{rev}, \text{da}_1, \text{da}_2, \text{arg}_2, \text{arg}_3 \}; \text{dec}_3 = \text{pro}.$

Note that the argument *pro* in the prior case, da_1 , can be strengthened in the current situation: instead of supporting *g* with merely *b*, support *g* with all of *b* & *e* & *f*. This alone does not suffice to establish *a*, however. *Pro* needs to cite the case in which *b*, a *prima facie* reason for *a*, was decided to be more important than *c*, a *prima facie* reason for *not-a*. Among the object-level arguments in ArgRec_3 , $\{ \text{da}_1, \text{da}_2, \text{arg}_2, \text{and } \text{arg}_3 \}$, *a* is not warranted. The important point is that da_1 was judged more important than da_2 in the past, and arg_3 prevents arg_2 from disrupting the past's bearing on the present.

IV. Related Work and Conclusions.

A *ratio decidendi* is an essential part of a case. To cite a case without attention to the *ratio* is to make analogies on surface similarities. Casually determined similarities may not actually be relevant to the past decision or its subsequent use. Similar remarks could be made about the application of rules without regard to the principles on which the rules are based: without regard for *ratio legis*.

There may be some latitude in the use of cases. In some protocols, whoever cites a case must produce the rationale. In other protocols, it is the responsibility of the opposition to raise any problem regarding rationale. Here, the assumption is that *ratio decidendi* need not always be given when a case is cited as part of an argument. The assumption is that the use of the case is appropriate. The benefits of such an assumption are like the benefits of any protocol that permits simpler argument moves. Unless the use of the case or the use of the rule is disputed, the assumption is that the use is appropriate. The same is true for rules. What is important is a mechanism in the protocol for permitting an objection to the use of a rule or case, when a party to the dispute elects to do so.

Rationales, especially *r*-rationales and *d*-rationales, illuminate why Ashley can make distinctions among cases [1989]. In our work, rules are extracted from cases. In any rule derived in this way, the sharing of important properties with the precedent is reason for sharing the decision of the precedent. Counterarguments are possible only by finding reasons for the opposite decision. Ashley and our improved analysis of *r*-rationales permit more sophisticated counterarguing. The opposition can simply cite properties that the present case does not share with the precedent. This kind of distinction becomes possible when the *r*-rationale or *d*-rationale of the case is available for examination.

Berman and Hafner recently discussed the telology of rules [1993]. The rationales of the rules they discuss are more complicated than the kinds discussed here. Their rationales involve a fundamentally different kind of reasoning: the reasoning about compromises in policy-making. The present work is less ambitious in scope and more ambitious in formality. It remains to be seen how many rationales can be expressed as compilation rationales of the five kinds explored here. Future work must surely be directed at representing the most important rationales in a particular legal domain. We do not expect that the full range of principles discussed for instance in Hart [1961], Dworkin [1985], or Peczenik [1989] could be accommodated with compilation rationales. The rationales chosen for investigation here are just the ones most amenable to treatment in the

existing model of argument and disputation. Branting [1993] is a formal analysis of rationales that does not rely on argument systems.

Prakken briefly discusses the possibility of modeling principle and purpose [1993]. He follows Gardner [1987] by taking most of the principle and purpose to be reflected in the matching of past cases to present case: i.e., in the extraction of rules from cases. “In this way it is possible to account for the defeasibility of legal rules caused by principle and purpose without having to complicate the formal model too much.” Of course, the matching of cases must be performed on the essential aspects of the case. It was precisely the delineation of the essential from the inessential that led us to investigate explicit representation of rationales. Ashley [1989], for example, assumes that only the important aspects of cases are formalized in the first place.

Prakken, noting agreement with Berman and Hafner, notes that reasoning with rationales will be unavoidably meta-level reasoning. Applying the model will determine whether the added representational power is worth the complication of the model to include meta-reasoning.

Formality plays an important role in some of AI’s interpretations of legal reasoning. Once the importance of rationales is allowed, the questions arise: what kinds of rationales? represented in what way? introduced according to what protocol? General studies of argument, such as Toulmin’s, have provided room for criticising the backing of a rule, the grounds on which it was adopted. But no prior work known to us has attempted to present details of this knowledge, their structures, and the processes in which they participate. To future automatons of rule-based and case-based legal reasoning, we hope studies of this kind will be useful.

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TYPESETTER'S ASSISTANCE:

Examples as contiguous figures, if you just want to cut and paste.

Example. A dialogue without a rationale.

1. pro: $\text{arg}_1 = \langle \{ b \>-- a \},$ i.e., b is reason for a;
b! $\therefore a \rangle$ i.e., b is asserted; therefore a.

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con: $\text{arg}_2 = \langle \{ c \>-- d,$ i.e., c is reason for d;
d & b $\>-- \text{not-a} \},$ i.e., d and b are joint reason for not-a;
c! b! $\therefore \text{not-a} \rangle$ i.e., c and b; therefore not-a.

$\text{ArgRec}_2 = \{ \text{arg}_1, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

3. pro: $\text{arg}_3 = \langle \{ c \& e \>-- \text{not-d} \},$ i.e., c and e are joint reason for not-d;
c! e! $\therefore \text{not-d} \rangle$ i.e., c and e; therefore not-d.

$\text{ArgRec}_3 = \{ \text{arg}_1, \text{arg}_2, \text{arg}_3 \}; \text{dec}_3 = \text{pro}.$

Here, arguments are 2-tuples (a set of rules paired with a conclusion). We have embellished the second argument. Instead of just giving the conclusion of the argument, we give the evidence on which the argument rests (marked with an exclamation point) and write “ \therefore ” to separate the conclusion. Rules are given in their sentential form, $a \>-- b$ instead of using the pair $\langle a, b \rangle$, and quotes are suppressed. Note that we use “ $=$ ” to give a name to a structure. This is not to be confused with an assertion.

At move 4, it is con's turn to move and opinion favors pro: arg_3 defeats arg_2 , thus reinstating arg_1 . If con cannot move at this point, pro wins.

Example. A c-rationale.

1. pro: $\text{arg}_1 = \langle \{ b \dashrightarrow a \},$
 $b! \therefore a \rangle$ “a because of b.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con: “your argument is really:”

2.1. $\text{c-rat}_1(b \dashrightarrow a) = \langle \{ b \dashrightarrow c,$
 $c \dashrightarrow a \},;$
 $b! \therefore a \rangle$ “c because of b, a because of c.”

2.2. $\text{Attack} \langle b \dashrightarrow a, \text{arg}_1 \rangle$ “because that is the rationale of
the rule in your argument, and now
here is my counterargument:”

2.3. $\text{arg}_2 = \langle \{ b \& d \dashrightarrow \text{not-c} \},$
 $b! d! \therefore \text{not-c} \rangle$ “not-c because of b and d.”

Con states a rationale for a rule, which presumably is the legitimate member of $R(b \dashrightarrow a)$. Con must state the argument being attacked. The effect is to replace the compressed argument with the uncompressed, but this is still not a sufficient response for con. So con provides arg_2 , an attack on the uncompressed argument, the *raison d’etre* for stating the rule’s rationale. @₂. Note $\text{pro} = \text{user}(b \dashrightarrow a, \text{arg}_1)$. $\text{con} = \text{attacker}(b \dashrightarrow a, \text{arg}_1)$.

$\text{ArgRec}_2 = \{ \text{arg}_1 \text{rev}, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

where

$\text{arg}_1 \text{rev} = \text{subst}(\text{arg}_1, b \dashrightarrow a, \text{c-rat}_1(b \dashrightarrow a))$

3. pro: $\text{arg}_3 = \langle \{ b \& d \& e \dashrightarrow c,$
 $c \dashrightarrow a \},$
 $b! d! e! \therefore a \rangle$ “ok, here is a new argument for c.”
“c because of b, d, and e; a because
of c.”

Pro provides a new argument for a, arg_3 , that is undefeated in ArgRec_3 . The new argument for c is specific enough to defeat arg_2 , which would reinstate arg_1 , but arg_1 has been flushed from the ArgRec in favor of $\text{arg}_1 \text{rev}$.

$\text{ArgRec}_3 = \text{ArgRec}_2 \cup \{ \text{arg}_3 \}; \text{dec}_3 = \text{pro}.$

Example. An r-rationale with maxcon-undef attacked.

1. pro: $\text{arg}_1 = \langle \{ b \ \& \ c \} \text{--} a \rangle,$
 $b! \ c! \ \therefore \ a \rangle$

“in virtue of b and c, and precedent, my claim is a.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con:

“that prior decision weighed two arguments, where the argument based on b, mediated by d was judged better than the argument based on c.”

2.1. $\text{r-rat}_1(b \ \& \ c \]\text{--} a) =$
 $\langle \{ da_1, da_2 \}, \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \}),$
 $\text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \}) \text{--} (b \ \& \ c \]\text{--} a) \rangle$

where

$da_1 = \langle \{ b \ \>\text{--} d$
 $d \ \>\text{--} a \},$
 $b! \ \therefore \ a \rangle,$

$da_2 = \langle \{ c \ \>\text{--} \text{not-a} \},$
 $c! \ \therefore \ \text{not-a} \rangle$

2.2. $\text{Attack} \langle b \ \& \ c \]\text{--} a, \text{arg}_1 \rangle$

2.3. $\text{arg}_1\text{rev} = \langle$
 $\{ \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \}) \text{--} (b \ \& \ c \]\text{--} a) \},$
 $b! \ c! \ \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \})! \ \therefore \ a \rangle$

“in order to defend the decision, you must at least be able to argue for a, and the opposition must have no better case than before.”

2.4. $\text{arg}_2 = \langle \{ e \ \>\text{--} f,$
 $f \ \>\text{--} \text{not-a} \},$
 $e! \ \therefore \ \text{not-a} \rangle$

“but here is a new argument that adds to con’s case in a non-trivial way and requires a re-weighing of arguments.”

2.5. $\text{not}(\text{maxcon-undef}(\{ da_2 \}))$

$\text{ArgRec}_2 = \{ \text{arg}_1\text{rev}, da_1, da_2, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

Note that among da_1 and da_2 , neither is defeating. The opinion in this case is significant because it creates a preference that cannot be found in $\text{extord}(\text{Args}(L, \{\}, E))$ or in defeat, that is, among the syntactically determinable or externally supplied orderings among arguments. The adjudicator chose between competing arguments that could not be decided purely on form. Con revises pro's argument arg_1 by producing arg_1rev which exposes the meta-reasoning. Then arg_2 is produced, which is an additional argument con, and is undefeated. So 2.5 is assertible, $\text{not}(\text{maxcon-undef}(\{da_2\}))$, which is one of the requirements for using the precedent.

Example. An r-rationale with minpro-undef attacked.

1. pro: $\text{arg}_1 = \langle \{ b \ \& \ c \]\text{-} a \},$
 $b! \ c! \ \therefore \ a \rangle$

“In virtue of b and c, and precedent, my claim is a.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con:

“that prior decision weighed two arguments, where the argument based on b, mediated by g was judged better than the argument based on c, mediated by d.”

2.1. $\text{r-rat}_1(b \ \& \ c \]\text{-} a) =$
 $\langle \{ \text{da}_1, \text{da}_2 \}, \text{minpro-undef}(\{ \text{da}_1 \}) \ \& \ \text{maxcon-undef}(\{ \text{da}_2 \}),$
 $\text{minpro-undef}(\{ \text{da}_1 \}) \ \& \ \text{maxcon-undef}(\{ \text{da}_2 \}) \ \>\text{-} (b \ \& \ c \]\text{-} a) \ \>$

where

$\text{da}_1 = \langle \{ b \ \>\text{-} g,$
 $g \ \>\text{-} a \},$
 $b! \ \therefore \ a \rangle,$

$\text{da}_2 = \langle \{ c \ \>\text{-} d,$
 $d \ \>\text{-} \text{not-}a \},$
 $c! \ \therefore \ \text{not-}a \rangle$

2.2. $\text{Attack} \langle b \ \& \ c \]\text{-} a, \text{arg}_1 \ \>$

2.3. $\text{arg}_1\text{rev} = \langle$
 $\{ \text{minpro-undef}(\{ \text{da}_1 \}) \ \& \ \text{maxcon-undef}(\{ \text{da}_2 \}) \ \>\text{-} (b \ \& \ c \]\text{-} a) \ \},$
 $b! \ c! \ \text{minpro-undef}(\{ \text{da}_1 \}) \ \& \ \text{maxcon-undef}(\{ \text{da}_2 \})! \ \therefore \ a \ \>$

“in order to defend the decision, you must at least be able to argue for a, and the opposition must have no better case than before.”

2.4. $\text{arg}_2 = \langle \{ b \ \& \ e \ \>\text{-} \text{not-}g \},$
 $b! \ e! \ \therefore \ \text{not-}g \ \>$

“but here is a new argument”

2.5. $\text{not}(\text{minpro-undef}(\{ \text{da}_1 \}))$

“which alters the status of the argument which, at minimum, must be possible for pro, in order to use the results of the prior case.”

$\text{ArgRec}_2 = \{ \text{arg}_1\text{rev}, \text{da}_1, \text{da}_2, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

Again, among da_1 and da_2 , neither is defeating, so the opinion in this case is significant. Again, the adjudicator chose between competing arguments that could not be decided purely on form. Con revises pro's argument arg_1 by producing arg_1rev which exposes the meta-reasoning as before. This time, arg_2 is produced, which defeats an essential part of the case, one of the minpro-undef arguments (in fact, the only minpro-undef argument), da_1 . So 2.5 is assertible, $not(minpro-undef(\{da_1\}))$, which again is a requirements for using the precedent.

Example. An f-rationale attacked for fit and redeemed in larger context.

1. pro: $\text{arg}_1 = \langle \{ b \>-- a \},$
 $b! \therefore a \rangle$ “a, because of b.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con:

- 2.1. $\text{f-rat}_2(b \>-- a) = \langle \text{Cases}_1, \text{Context}_1, \text{fit-r}(b \>-- a, \text{Cases}_1, \text{Context}_1) \rangle$
 “the rationale for the rule you use is that it fits two main cases.”

where

$\text{Cases}_1 = \{ \text{case}_1, \text{case}_2 \}$
 $\text{case}_1 = \langle \{ b, d \}, a \rangle$
 $\text{case}_2 = \langle \{ \text{not-}b \}, \text{not-}a \rangle$
 $\text{Context}_1 = \{ \}$

- 2.2. Attack $\langle b \>-- a, \text{arg}_1 \rangle$

- 2.3. $\text{arg}_1\text{rev} = \langle$
 $\{ \text{ok}(\text{Cases}_1) \ \& \ \text{fit-r}(b \>-- a, \text{Cases}_1, \text{Context}_1) \>-- (b \>-- a) \},$
 $\text{ok}(\text{Cases}_1)! \text{fit-r}(b \>-- a, \text{Cases}_1, \text{Context}_1)! b! \therefore a \rangle$
 “your argument depends on those prior decisions being ok, and the fit of the rule to those cases.”

- 2.4. $\text{arg}_2 = \langle$
 $\{ \text{ok}(\text{Cases}_2) \ \& \ \text{not-fit-r}(b \>-- a, \text{Cases}_2, \text{Context}_1) \>-- \text{not}(b \>-- a) \},$
 $\text{ok}(\text{Cases}_2)! \text{fit-r}(b \>-- a, \text{Cases}_2, \text{Context}_1)! \therefore \text{not}(b \>-- a) \rangle$
 “but there is a third important case that the rule does not fit.”

where

$\text{Cases}_2 = \{ \text{case}_1, \text{case}_2, \text{case}_3 \}$
 $\text{case}_3 = \langle b, c, \text{not-}a \rangle$

$\text{ArgRec}_2 = \{ \text{arg}_1\text{rev}, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

Con has added a case, case₃, which the rule, b >-- a, does not fit in Context₁. Pro responds by exhibiting a larger context which accounts for the alleged deviation.

3. pro: arg₃ = <
 { ok(Cases₂) & fit-r(b >-- a , Cases₁, Context₂) >-- (b >-- a) },
 ok(Cases₂)! fit-r(b >-- a , Cases₂, Context₂)! b! ∴ a >
 “perhaps the case is important,
 but it is subsumed under a different
 rule in the statutory context.”

where

Context₂ = { b & c >-- not-a }

ArgRec₃ = { arg₁rev, arg₂, arg₃ }; dec₃ = pro.

Example. Deepening an r-rationale-based attack.

1. pro: $\text{arg}_1 = \langle \{ b \ \& \ c \} \text{--} a \rangle,$
 $b! \ c! \ \therefore \ a \rangle$

“in virtue of b and c, and precedent, my claim is a.”

$\text{ArgRec}_1 = \{ \text{arg}_1 \}; \text{dec}_1 = \text{pro}.$

2. con:

“that prior decision weighed two arguments, where the argument based on b, mediated by g was judged better than the argument based on c, mediated by d.”

2.1. $\text{r-rat}_1(b \ \& \ c \]\text{--} a) =$
 $\langle \{ da_1, da_2 \}, \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \}),$
 $\text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \}) \ \>\text{--} (b \ \& \ c \]\text{-} a) \ \>$

where

$da_1 = \langle \{ b \ \>\text{--} g,$
 $g \ \>\text{--} a \},$
 $b! \ \therefore \ a \rangle,$

$da_2 = \langle \{ c \ \>\text{--} d,$
 $d \ \>\text{--} \text{not-}a \},$
 $c! \ \therefore \ \text{not-}a \rangle$

2.2. $\text{Attack} \langle b \ \& \ c \]\text{--} a, \text{arg}_1 \ \>$

2.3. $\text{arg}_1\text{rev} = \langle$
 $\{ \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \}) \ \>\text{--} (b \ \& \ c \]\text{-} a) \ \},$
 $b! \ c! \ \text{minpro-undef}(\{ da_1 \}) \ \& \ \text{maxcon-undef}(\{ da_2 \})! \ \therefore \ a \ \>$

“in order to defend the decision, you must at least be able to argue for a, and the opposition must have no better case than before.”

2.4. $\text{arg}_2 = \langle \{ b \ \& \ e \ \>\text{--} \text{not-}g \},$
 $b! \ e! \ \therefore \ \text{not-}g \ \>$

“but here is a new argument”

2.5. $\text{not}(\text{minpro-undef}(\{ da_1 \}))$

“which alters the status of the argument which, at minimum, must be possible for pro, in order to use the results of the prior case.”

$\text{ArgRec}_2 = \{ \text{arg}_1\text{rev}, da_1, da_2, \text{arg}_2 \}; \text{dec}_2 = \text{none}.$

This is as before: arg_2 is produced, which defeats an argument that must be minpro-undef. Now pro responds by attacking arg_2 and ejecting it from $ArgRec_3$.

3. pro:

3.1. $arg_3 = \langle \{ b \ \& \ e \ \& \ f \} \dashv\vdash g \rangle,$
 $b! \ e! \ f! \ \therefore \ g$

“ok, but your argument can be defeated”

3.2. minpro-undef($\{da_1\}$)

“which restores the minimum argument pro”

$ArgRec_3 = \{ arg_1rev, da_1, da_2, arg_2, arg_3 \}; dec_3 = pro.$

Note that the argument pro in the prior case, da_1 , can be strengthened in the current situation: instead of supporting g with merely b , support g with all of $b \ \& \ e \ \& \ f$. This alone does not suffice to establish a , however. Pro needs to cite the case in which b , a prima facie reason for a , was decided to be more important than c , a prima facie reason for not- a . Among the object-level arguments in $ArgRec_3$, $\{ da_1, da_2, arg_2, \text{ and } arg_3 \}$, a is not warranted. The important point is that da_1 was judged more important than da_2 in the past, and arg_3 prevents arg_2 from disrupting the past’s bearing on the present.