A Mathematical Comment on the Fundamental Difference between Legal Theory Formation and Scientific Theory Formation

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May 30, 2005

1 Introduction

test

2 Scientific Theory Formation: Model I

A set of n cases is $C = \{< c_i, o_i > | i = 1, .., n\}$. Each circumstance, c_i, is a vector in $\Re^k$. Either all observations, o_i are real-valued, in $\Re$, or all are boolean, in $\{0, 1\}$.

A parameterized function, f, with parameters $p \in \Re^m$ is chosen to fit the cases. f is a mapping $\Re^k \rightarrow \{0, 1\}$ or $\Re^k \rightarrow \Re$, depending on the values of o_i.

The error of f's fit with C is defined as a function of p, $E(f_p(C))$, or $E(p)$ for short:

$$E(p) = \Sigma_{i=1,..,n}(f(c_i) \neq o_i)$$

is just the number of disagreements between the value of the function and the observed value. This makes most sense for boolean observations. For real-valued observations, a norm is often used, such as squared error:

$$E(p) = \Sigma_{i=1,..,n}\|f(c_i) - o_i\| \ .$$

The scientific theory formation problem is to find $p* \in \Phi$ to minimize the error:

$$argmin_{p \in \Phi} E(f_p(C)) \ .$$

For example, for $k = 1$, linear regression is the most basic curve-fitting, on $p =< m, b >$, which is a kind of theory-formation.

3 Scientific Theory Formation: Model II

A simplicity ordering on functional forms, $\prec$, permits one function, f to be compared to another, g on grounds of simplicity. For f_{p_1}, where p_1 has m dimensions or *degrees of freedom*, any g_{p_2} where p_2 has $m - 1$ dimensions will be simpler.