The Strategy of Rotten Tomatoes

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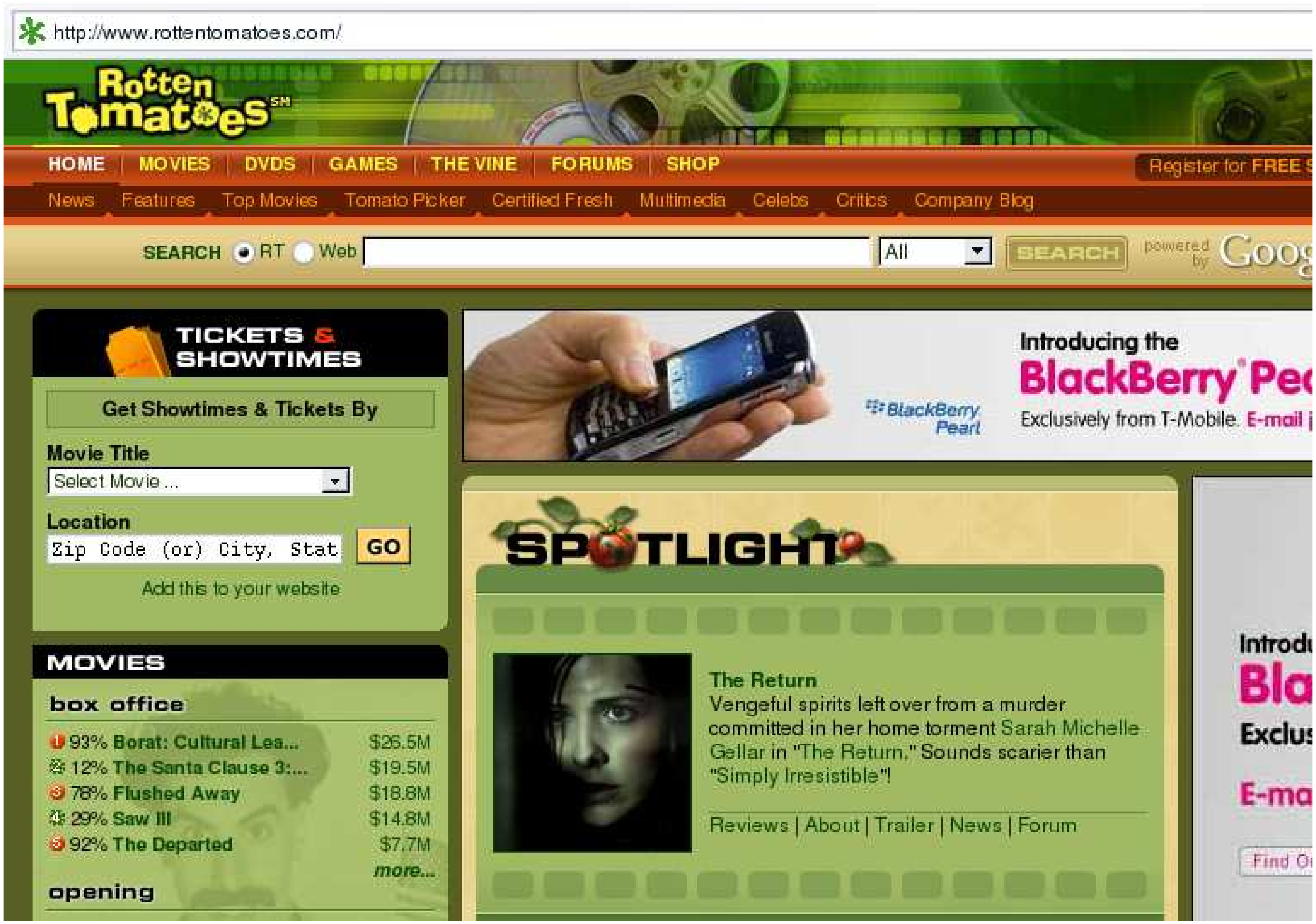
rottentomatoes.com



Tomatometer

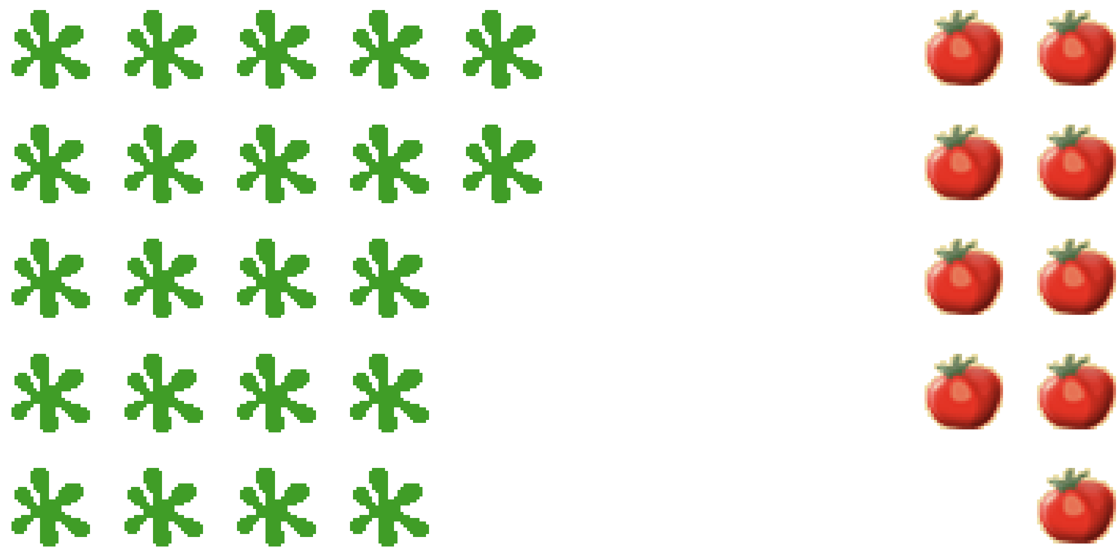
Saw III: 29%

Borat: 93%

Tomatometer

31 "votes" (from reviews) for *Saw III*:

$\vec{v} =$



22 rotten 9 fresh

$$t = \frac{9}{31} = 29\%$$

RT's newest critic

Borat Sagdiyev, Kazakh Ministry of Information

"It is nice!"

Borat's first assignment

Flushed Away

Borat says, "I like . . . $\overbrace{\text{70\%}}^{r_{\text{Borat}} = .7}$."

Should he vote it $\underbrace{\text{fresh}}_{v_{\text{Borat}} = 1}$ or $\underbrace{\text{rotten}}_{v_{\text{Borat}} = 0}$?

How should Borat vote?

26 votes for *Flushed Away* so far:

$\vec{v} =$

6 rotten 20 fresh

$$t = \frac{20}{26} = 77\%$$

How should Borat vote?

if Borat votes it "fresh":

$\vec{v} =$

6 rotten 21 fresh

$$t = \frac{21}{27} = 78\%$$

How should Borat vote?

if Borat votes it “rotten”:

$\vec{v} =$

7 rotten 20 fresh

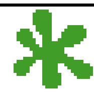
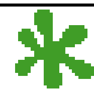
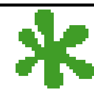
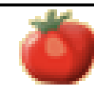
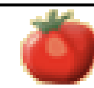
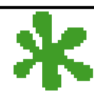
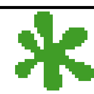
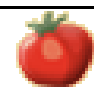
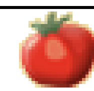
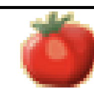
$$t = \frac{20}{27} = 74\%$$

Borat decides

He votes rotten.

- but the other critics are just as smart as Borat!
- if every critic manipulated like Borat, would chaos ensue?

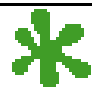
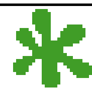
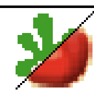
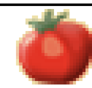
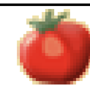
Chaos, of a sort . . .

			Borat			
$\vec{r} =$	0	.2	.5	.7	1	
$\vec{v} =$						$t = \bar{v} = .4$
$\vec{v} =$						$t = \bar{v} = .6$

if Borat keeps trying to realize .5, a cycle results

Chaos avoided

but if partial votes are allowed:

			Borat			
$\vec{r} =$	0	.2	.5	.7	1	
$\vec{v} =$						$t = \bar{v} = .5$
	(0)	(0)	(.5)	(1)	(1)	

equilibrium is reached!

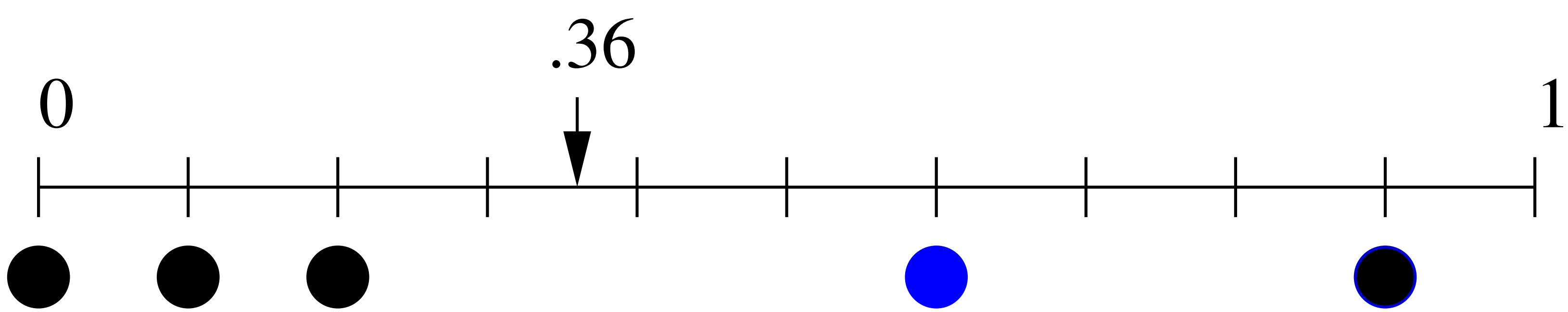
so, allow $(\forall i)\ 0 \le v_i \le 1$
(like metacritic.com)

One approach: Average

$$\vec{r} = [0, .1, .2, .6, .9]$$

$$\vec{v} = [0, .1, .2, .6, .9]$$

outcome $t = \bar{v} = .36$

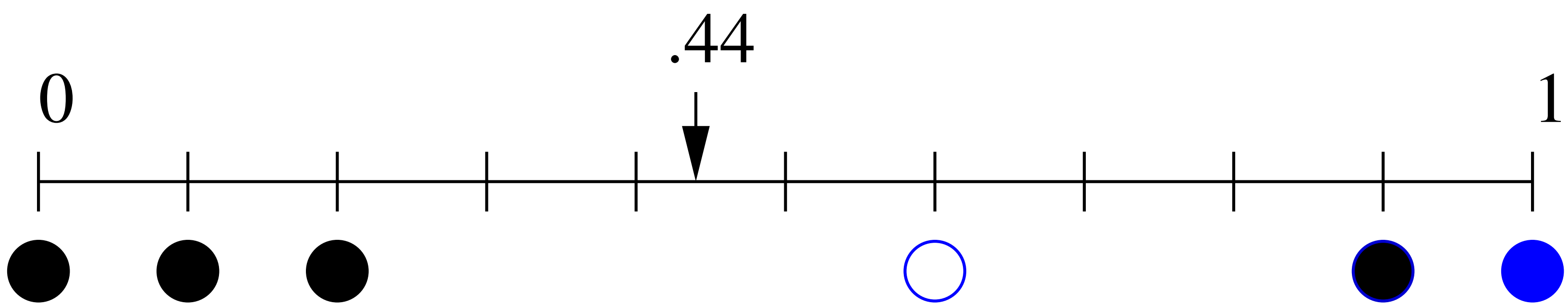


One approach: Average

$$\vec{r} = [0, .1, .2, .6, .9]$$

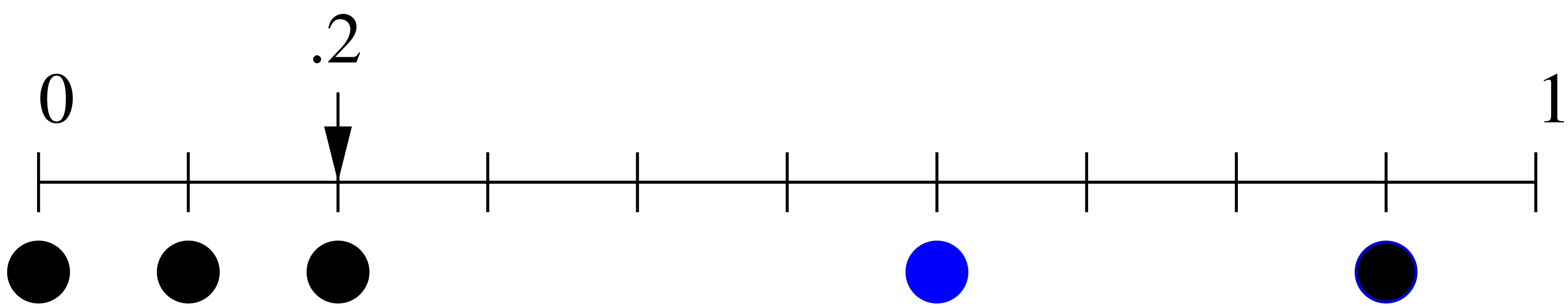
$$\vec{v} = [0, .1, .2, 1, .9]$$

$$\text{outcome } t = \bar{v} = .44$$



Another approach: Median

$$\vec{r} = [0, .1, .2, .6, .9]$$
$$\vec{v} = [0, .1, .2, .6, .9]$$
$$\text{outcome } t = \tilde{v} = .2$$



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$$\vec{r} = [0, .1, .2, .6, .9]$$
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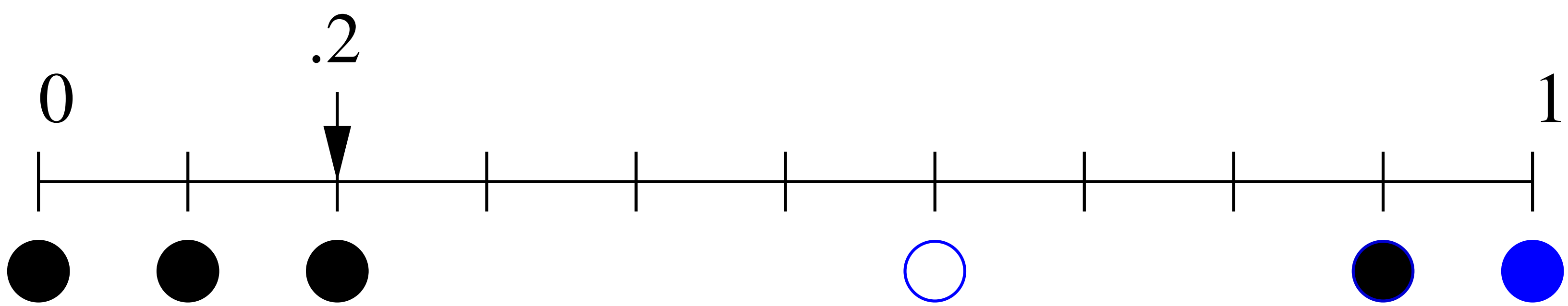

Another approach: Median

- nonmanipulable
  - voter i cannot obtain a better result by voting $v_i \neq r_i$
  - if $\tilde{v} < v_i$, increasing v_i will not change $\tilde{v}$
  - if $\tilde{v} > v_i$, decreasing v_i will not change $\tilde{v}$
- allows tyranny by a majority
  - $\vec{v} = [0, 0, 1, 1, 1]$
  - $t = \tilde{v} = 1$
  - no concession to the 0-voters
  - so not ideal for Rotten Tomatoes—what now?

Formalizing Borat's thinking

- for $1 \leq i \leq n$, voter i should choose v_i to move outcome as close to r_i as possible
- choosing $v_i = r_i n - \sum_{j \neq i} v_j$ would give $\bar{v} = r_i$
- optimal vote is $v_i = \min(\max(r_i n - \sum_{j \neq i} v_j, 0), 1)$
- after voter i uses this strategy, one of these is true:

1. $\bar{v} < r_i$ and $v_i = 1$
2. $\bar{v} = r_i$
3. $\bar{v} > r_i$ and $v_i = 0$

What happens at equilibrium?

- the optimal strategy recommends that no voter change
- so $(\forall i)\ \bar{v} < r_i \longrightarrow v_i = 1$
- and $(\forall i)\ \bar{v} > r_i \longrightarrow v_i = 0$
- (equivalently, $(\forall i)\ v_i \neq 0 \longrightarrow \bar{v} \leq r_i$)
- therefore any average at equilibrium must satisfy two equations:

  (A) $\bar{v}n \leq |\{i : \bar{v} \leq r_i\}|$

  (B) $|\{i : \bar{v} < r_i\}| \leq \bar{v}n$
- but will an equilibrium always exist? (conjecture: yes)

Multiple equilibria possible

$$\vec{r} = [.2, .3, .5, .5, .8]$$

$$\vec{v} = [0, 0, .5, 1, 1]$$

$$\vec{v} = [0, 0, .6, .9, 1]$$

$$\vec{v} = [0, 0, .75, .75, 1]$$

in each case:

$$t = \bar{v} = .5$$

Only one equilibrium average

$$A(\phi) \equiv \quad \phi n \leq |\{i : \phi \leq r_i\}|$$
$$B(\phi) \equiv \quad |\{i : \phi < r_i\}| \leq \phi n$$

- theorem:

$$\boxed{A(\phi_1) \wedge B(\phi_1) \wedge A(\phi_2) \wedge B(\phi_2) \longrightarrow \phi_1 = \phi_2}$$

- proof considers two symmetric cases:
  - assume $\phi_1 < \phi_2$
  - assume $\phi_2 < \phi_1$
- each leads to a contradiction

Only one equilibrium average

case 1: $\phi_1 < \phi_2$

Only one equilibrium average

case 1: $\phi_1 < \phi_2$

- $(\forall i)\ \phi_2 \leq r_i \longrightarrow \phi_1 < r_i$

Only one equilibrium average

case 1: $\phi_1 < \phi_2$

- $(\forall i)\ \phi_2 \leq r_i \longrightarrow \phi_1 < r_i$
- $\{i : \phi_2 \leq r_i\} \subseteq \{i : \phi_1 < r_i\}$

Only one equilibrium average

case 1: $\phi_1 < \phi_2$

- $(\forall i)\ \phi_2 \le r_i \longrightarrow \phi_1 < r_i$
- $\{i : \phi_2 \le r_i\} \subseteq \{i : \phi_1 < r_i\}$
- $|\{i : \phi_2 \le r_i\}| \le |\{i : \phi_1 < r_i\}|$

Only one equilibrium average

case 1: $\phi_1 < \phi_2$

- $(\forall i)\ \phi_2 \leq r_i \longrightarrow \phi_1 < r_i$
- $\{i : \phi_2 \leq r_i\} \subseteq \{i : \phi_1 < r_i\}$
- $|\{i : \phi_2 \leq r_i\}| \leq |\{i : \phi_1 < r_i\}|$
- $\phi_2 n \leq |\{i : \phi_2 \leq r_i\}| \qquad (A(\phi_2))$

Only one equilibrium average

case 1: $\phi_1 < \phi_2$

- $(\forall i)\ \phi_2 \leq r_i \longrightarrow \phi_1 < r_i$
- $\{i : \phi_2 \leq r_i\} \subseteq \{i : \phi_1 < r_i\}$
- $|\{i : \phi_2 \leq r_i\}| \leq |\{i : \phi_1 < r_i\}|$
- $\phi_2 n \leq |\{i : \phi_2 \leq r_i\}| \qquad (A(\phi_2))$
- $|\{i : \phi_1 < r_i\}| \leq \phi_1 n \qquad (B(\phi_1))$

Only one equilibrium average

case 1: $\phi_1 < \phi_2$

- $(\forall i)\ \phi_2 \le r_i \longrightarrow \phi_1 < r_i$
- $\{i : \phi_2 \le r_i\} \subseteq \{i : \phi_1 < r_i\}$
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- $\phi_2 n \le |\{i : \phi_2 \le r_i\}|$ $\qquad (A(\phi_2))$
- $|\{i : \phi_1 < r_i\}| \le \phi_1 n$ $\qquad (B(\phi_1))$
- $\phi_2 n \le |\{i : \phi_2 \le r_i\}| \le |\{i : \phi_1 < r_i\}| \le \phi_1 n$

Only one equilibrium average

case 1: $\phi_1 < \phi_2$

- $(\forall i)\ \phi_2 \leq r_i \longrightarrow \phi_1 < r_i$
- $\{i : \phi_2 \leq r_i\} \subseteq \{i : \phi_1 < r_i\}$
- $|\{i : \phi_2 \leq r_i\}| \leq |\{i : \phi_1 < r_i\}|$
- $\phi_2 n \leq |\{i : \phi_2 \leq r_i\}| \qquad (A(\phi_2))$
- $|\{i : \phi_1 < r_i\}| \leq \phi_1 n \qquad (B(\phi_1))$
- $\phi_2 n \leq |\{i : \phi_2 \leq r_i\}| \leq |\{i : \phi_1 < r_i\}| \leq \phi_1 n$
- $\phi_2 n \leq \phi_1 n$

Only one equilibrium average

case 1: $\phi_1 < \phi_2$

- $(\forall i)\ \phi_2 \le r_i \longrightarrow \phi_1 < r_i$
- $\{i : \phi_2 \le r_i\} \subseteq \{i : \phi_1 < r_i\}$
- $|\{i : \phi_2 \le r_i\}| \le |\{i : \phi_1 < r_i\}|$
- $\phi_2 n \le |\{i : \phi_2 \le r_i\}|$ $(A(\phi_2))$
- $|\{i : \phi_1 < r_i\}| \le \phi_1 n$ $(B(\phi_1))$
- $\phi_2 n \le |\{i : \phi_2 \le r_i\}| \le |\{i : \phi_1 < r_i\}| \le \phi_1 n$
- $\phi_2 n \le \phi_1 n$
- $\phi_2 \le \phi_1$ (contradicting $\phi_1 < \phi_2$)

Only one equilibrium average

- case 1 shows that $\phi_1 \not< \phi_2$
- case 2 is symmetrical and shows that $\phi_2 \not< \phi_1$
- therefore $\phi_1 = \phi_2$ $\square$

therefore, given $\vec{r}$, the average at equilibrium is unique

Declared-Strategy Voting

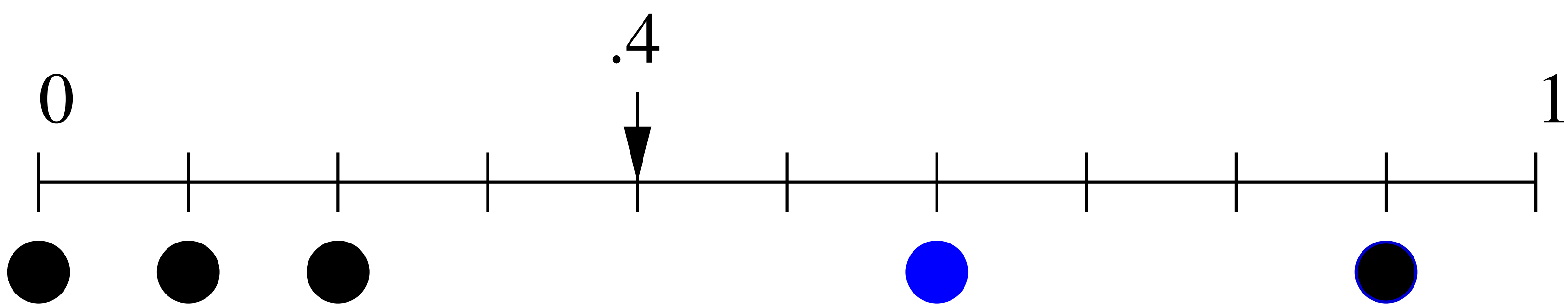
- move the manipulation from voter into voting system?
- DSV [Cranor & Cytron '96] treats voters' input as sincere preferences and strategizes for them
- DSV aims to encourage sincerity by embracing manipulation
- apply DSV to Average
  - $\bar{v}$ at equilibrium can be defined as a function $f(\vec{r})$
  - $f(\vec{v})$ gives a new system: Equilibrium Average

Equilibrium Average

$$\vec{r} = [0, .1, .2, .6, .9]$$

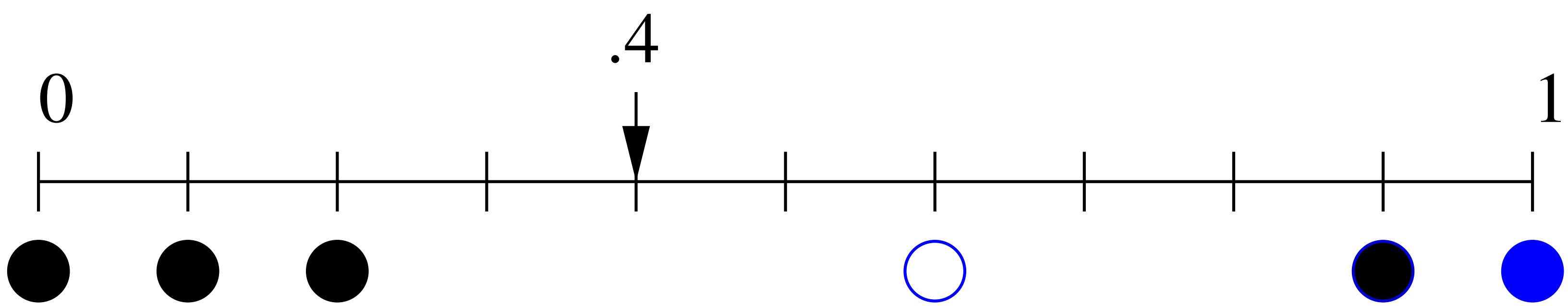
$$\vec{v} = [0, .1, .2, .6, .9]$$

$$\text{outcome } t = \hat{v} = .4$$



Equilibrium Average

$$\vec{r} = [0, .1, .2, .6, .9]$$
$$\vec{v} = [0, .1, .2, 1, .9]$$
$$\text{outcome } t = \hat{v} = .4$$



Equilibrium Average (cont.)

- conjecture: Equilibrium Average is nonmanipulable
  - intuitive reasoning is similar to case of Median: if $\hat{v} < v_i$, increasing v_i will not change $\hat{v}$
- no tyranny by a majority
  - $\vec{v} = [0, 0, 1, 1, 1]$
  - $t = \hat{v} = .6$

Contributions

- applied DSV framework to a new kind of voting system
- found rational strategy for Average
- showed that the strategy leads to unique outcomes
- defined a new system, Equilibrium Average, which
  - is a computable function
  - encourages sincerity

Future work

- finish proof that an Average equilibrium always exists
- finish proof that Equilibrium Average is nonmanipulable
- explore effects of DSV on other systems

[Cranor & Cytron '96]: Lorrie Cranor and Ron K. Cytron. "Towards an Information-Neutral Voting Scheme That Does Not Leave Too Much to Chance." Paper presented at the Midwest Political Science Association Annual Meeting, 18-20 April 1996

Thanks!

to

Ron Cytron, the DOC Group, Aaron Stump

and you, the attentive audience

Questions?

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