Fairness: How to Measure It Quantitatively?

Raj Jain
Professor of CIS

Raj Jain is now at
Washington University in Saint Louis
Jain@cse.wustl.edu
http://www.cse.wustl.edu/~jain/
Index of fairness

Why is it better than others?

References:


**Fairness**

- **Simple Definition:** *Equal* share of bottleneck

**Problem:** Some VC’s may be bottlenecked elsewhere

- **Next Definition:** Optimal Allocation or Equal fraction of optimal allocation

- **Example:** A scheme gives 100, 4, 10.5 Mbps when the optimal is 100, 40, 15 Mbps. How fair is it? 67%? 90%?
Proposal

- Actual allocation: \((A_1, A_2, \ldots, A_n)\)
- Use any criterion (e.g., max-min optimality) to find the optimal allocation \((O_1, O_2, \ldots, O_n)\)
- Relative allocation: \(x_i = \frac{A_i}{O_i}\)

Fairness = \(\frac{(\sum x_i)^2}{n \sum x_i^2}\)

Example: 100/100, 4/40, 10.5/15 \(\Rightarrow\) 1, 0.1, 0.9

Fairness = \(\frac{(1+0.1+0.9)^2}{3(1^2+0.1^2+0.9^2)} = \frac{2^2}{3(1+0.01+0.81)} = 0.73\)
Other Proposals

- Find the variance, standard deviation, or coefficient of variation
  
  Mean $\mu = (1+0.1+0.9)/3 = 0.67$
  
  Variance $\sigma^2 = (1/n)\Sigma(x_i-\mu)^2 = 0.16$
  
  Standard deviation $\sigma = 0.4$
  
  Coefficient of variation $= \sigma/\mu = 0.597$

- Find the distance from the optimal

  \[
  \text{Fairness} = \frac{\left[\Sigma (A_i-O_i)^2\right]^{1/2}}{\left[\Sigma O_i^2\right]^{1/2}} = \frac{[0^2+0.9^2+0.1^2]^{1/2}}{[1^2+1^2+1^2]^{1/2}} = 0.52
  \]

- Min/Max $= 0.1/1 = 0.1$
Fairness Index: Properties

- Applicable for any number of VCs, even \( n=2 \)
  Strictly speaking, variance not defined for small \( n \).

- Scale independent.
  Variance (Throughput) = \( 10 \text{ Mbps}^2 = 10^7 \text{ kbps}^2 \)
  Standard deviation (Throughput) = \( 10 \text{ Mbps} = 10^4 \text{ kbps} \)

- Bounded between 0 and 1 or 0 and 100%
  Variance, standard deviation, and Relative distance are not bounded.

- Direct relationship: Higher index \( \Rightarrow \) More Fair
  Higher variance \( \Rightarrow \) Less fair

- Continuous. Min/max is not continuous.
Fairness Index: Properties

- Intuitive:
  - For \((1, 0, 1)\) Index = \(2/3\)
  - For \(x_i = 1\), \(i=1,2,3,...,k\) 
    \[\text{Index} = k/n\]
  - If 80% of the users are treated fairly and 20% are starved, index = 80%
The following text be added to the baseline text.

The fairness will be quantified using the following formula:

\[
\text{Fairness} = \frac{(\sum x_i)^2}{n \sum x_i^2}
\]

where \( x_i = \text{ratio of actual throughput/optimal throughput} \).