Structure from Motion
Structure from Motion

- Given many images, compute camera parameters...
Camera calibration and triangulation

• Suppose we know 3D points
  – And have matches between these points and an image
  – How can we compute the camera parameters?

• Suppose we know camera parameters, each of which observes a point
  – How can we compute the 3D location of that point?
Structure from motion

• SfM solves both of these problems *at once*
• A kind of chicken-and-egg problem
  – (but solvable)
Structure from Motion

\[ \Pi_1 X_1 \sim p_{11} \]

\[
\begin{align*}
\text{minimize} & \quad g(R, T, X) \\
& \text{non-linear least squares}
\end{align*}
\]
Structure from motion

• Minimize sum of squared reprojection errors:

\[ g(X, R, T) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij} \cdot \left\| P(x_i, R_j, t_j) - \begin{bmatrix} u_{i,j} \\ v_{i,j} \end{bmatrix} \right\|^2 \]

indicator variable: is point \( i \) visible in image \( j \)?
Structure from motion

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*indicator variable:* is point \( i \) visible in image \( j \)?

- Minimizing this function is called *bundle adjustment*
  - Optimized using non-linear least squares, e.g. Levenberg-Marquardt
Structure from Motion

\[ \Pi_1 X_1 \sim p_{11} \]

\[ \text{minimize} \quad g(R, T, X) \]

\[ \text{non-linear least squares} \]
Non-linear Optimization

• How to find optimal R, T, X

\[
\text{minimize} \quad g(R, T, X)
\]
Non-linear Optimization

• How to find optimal $R, T, X$

$$\text{minimize } g(R, T, X)$$

**fminsearch**

Find minimum of unconstrained multivariable function using derivative-free method

Nonlinear programming solver that searches for the minimum of a problem specified by

$$\min_x f(x)$$

$f(x)$ is a function that returns a scalar, and $x$ is a vector or a matrix.

**Syntax**

```matlab
x = fminsearch(fun,x0)
```
Non-linear Optimization

• Need a good initialization!

```
\textbf{fminsearch}

Find minimum of unconstrained multivariable function using derivative-free method

Nonlinear programming solver that searches for the minimum of a problem specified by

\[
\min_x f(x)
\]

\(f(x)\) is a function that returns a scalar, and \(x\) is a vector or a matrix.

**Syntax**

\[x = \text{fminsearch}(\text{fun},x0)\]
```
Structure from motion

• Given many images, how can we
  a) figure out where they were all taken from?
  b) build a 3D model of the scene?
First step: how to get correspondence?

• Feature detection and matching
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]
Feature detection

Detect features using SIFT [Lowe, IJCV 2004]
Feature matching

Match features between each pair of images
Feature matching

Refine matching using RANSAC to estimate fundamental matrix between each pair
Image connectivity graph

(graph layout produced using the Graphviz toolkit: http://www.graphviz.org/)
Feature matching

Refine matching using RANSAC to estimate fundamental matrix between each pair

$K$ from EXIF
Feature matching

Refine matching using RANSAC to estimate fundamental matrix between each pair

$K$ from EXIF

Essential matrix $R, T$
Feature matching

Refine matching using RANSAC to estimate fundamental matrix between each pair

Essential matrix

R, T

Optimization
Feature matching

Refine matching using RANSAC to estimate fundamental matrix between each pair

Essential matrix

$R, T$

Optimization
Steps

• Pick a pair of images with lots of features
• Initialize intrinsics K from EXIF
• Solve Essential matrix
• Compute R and t
• While remaining images exist
  — Find an image with many feature matches with images in the model
  — RANSAC on feature matches to compute R/t
  — Non linear optimization to optimize everything
Problem size

• What are the variables?
• How many variables per camera?
• How many variables per point?

• Trevi Fountain collection
  466 input photos
  + > 100,000 3D points
  = very large optimization problem
Is SfM always uniquely solvable?
Is SfM always uniquely solvable?

- No - up to scaled-rigid transformation
SfM – Failure cases

- Necker reversal (if close to orthographic)
Structure from Motion – Failure cases

• Repetitive structures
Photo Tourism
Exploring photo collections in 3D

Noah Snavely  Steven M. Seitz  Richard Szeliski
University of Washington  Microsoft Research

SIGGRAPH 2006
Large-scale structure from motion
Libration

From Wikipedia, the free encyclopedia

Not to be confused with Liberation or Libation.

In astronomy libration (from the Latin verb librare "to balance, to sway", cf. libra "scales") refers to the various orbital conditions which make it possible to see more than 50% of the moon's surface over time, even though the front of the Moon is tidally locked to always face towards Earth. By extension, libration can also be used to describe the same phenomenon for other orbital bodies that are nominally locked to present the same face. As the orbital processes are repetitive, libration is manifested as a slow rocking back and forth (or up and down) of the face of the orbital body as viewed from the parent body, much like the rocking of a pair of scales about the point of balance.

In the specific case of the Moon's librations, this motion permits a terrestrial observer to see slightly differing halves of the Moon's surface at different times. This means that a total of 59% of the Moon's surface can be observed from Earth.

There are three types of libration:

- **Libration in longitude** is a consequence of the Moon's orbit around Earth being somewhat eccentric, so that the Moon's rotation sometimes leads and sometimes lags its orbital position.

- **Libration in latitude** is a consequence of the Moon's axis of rotation being slightly inclined to the normal to the plane of its orbit around Earth. Its origin is analogous to the way in which the seasons arise from Earth's revolution about the Sun.

- **Diurnal libration** is a small daily oscillation due to the Earth's rotation, which carries an observer first to one side and then to the other side of the straight line joining Earth's center to the Moon's center, allowing the observer to look first around one side of the Moon and then around the other. This is because the observer is on the surface of the Earth, not at its centre.
Questions?
SfM applications

• 3D modeling
• Surveying
• Robot navigation and mapmaking
• Visual effects...