Approaching Mergers and Acquisitions with Fair Division Algorithms

Abstract
In this paper, we detail our approach to quantify the continuous pricing errors in corporate mergers and acquisitions and suggest an algorithmic approach to more accurately price an M&A transaction. Qualitatively, our algorithm provides a logical path to prevent certain value destructive transactions and encourage transactions that add economic value. Quantitatively, it also outlines a methodology to ensure proportionality in each transaction, and propose a theoretical envy free solution. We analyze this problem within the scope of cake cutting theory, introduce cheating in the form of pricing discrepancy, and discuss how traditional fair division models fail. The ultimate goal of our analysis is to suggest a procedure with which bankers, target companies, and acquiring companies can value a possible merger systematically and realistically that some of the common causes of failed mergers and acquisitions can naturally be prevented.

1. Introduction

According to data compiled by Bloomberg, since Jan, 2000 there were 12,111 mergers and acquisitions deals on a yearly basis with an average of 30% premium attached to each deal. The total dollar amounts of the deals can be misleading due to currency and time value, but even so, the deals totaled to a modest $35.73 trillion, a 30% premium implies $10.7 trillion. The premium or discount attached to each transaction cannot be explained with a general concept because depending on the macroeconomic, industry and company specific situation, the deal must be assessed individually. However, the 30% average premium cannot be justified when 70% of the deals fail to achieve the projected synergies according to Matt H. Evans¹. That said, we decided to look into a certain type of mergers and acquisitions with certain restrictions where a cake cutting approach is applicable.

There are two major motivations for companies to take on M&A, strategic and financial, which can be further divided into horizontal, vertical, and conglomerate deals. Non-strategic M&As happen when the drivers of the transaction are to take advantage of the financial valuation of the company, to change the financial structure of the company, to massage financial statements, to fulfill managerial glut for empire, and to exit or liquidate due to current economic, industry specific, and company specific situations. There exist grey areas of the divider between a strategic and non-strategic M&A depending on how the transaction takes place and is carried out. However, for our purpose we will only define strategic vertical mergers as a possible candidate for the cake cutting approach.
Strategic horizontal M&A is a transaction where two companies operating in the same industry area with same or similar products merge or acquire one another to form a new company. There are three clear major drivers behind a vertical merger. The companies, when combined can cut significant amount of both fixed and variable costs depending on the characteristics of the business such as administration costs, R&D costs, overlapping human capital, and cost of capital. In addition, with a combined larger company follows a stronger buying and negotiation power with the suppliers and clients, the combined corporation will be able to negotiate a cheaper supply cost, and strategically target a wider customer base. Lastly, the combined companies may be able to supplement each other’s weaknesses and apply their unique strengths to more areas.

2. Simple Agreement

We designed a simple test using a mean searching algorithm to price a transaction. In this test, we replicated six high-level scenarios of M&A to ultimately compare the aggregate test results with the data compiled by Bloomberg.

Assumptions

With limited access to data, we made a number of assumptions and conditions to increase validity of our reasoning and applicability of our analysis.
- Price reflects all expectations and information from the market.
- All companies involved in the transaction have public financial statements.
- All transactions are priced by the target and the acquirer relative to the market price.
- All participants conduct valuation with respect to the market price.
- All transactions are strategically motivated.
- Involved parties want to benefit from the deal.

Notation

\[ R_1 = \frac{1}{y}, \text{ a random influence variable with guided variance} \]
\[ R_2 = \frac{1}{z}, \text{ a random influence variable with guided variance} \]
\[ K_1 \& K_2 = \text{upper and lower bound variables for } t(T) \text{ and } a(T) \]
\[ M(T) = \frac{1}{x}, \text{ market’s view of the target as a proportion of the merged company} \]
- M(T) is also a random variable between 0 and 1
\[ T(T) = M(T) + R_1, \text{ target’s view of itself as a proportion of the merged company} \]
- K1 < T(T) < 1
\[ A(T) = M(T) + R_2, \text{ acquirer’s view of the target as a proportion of the merged company} \]
- 0 < A(T) < K2

* Min(T(T)) <= Max(A(T)) We are assuming the deal occurs in the test scenarios
Implementation
The test is a simple Monte Carlo simulation of finding the average between $T(T)$ and $A(T)$. We implemented the simulation on six different scenarios.

Scenario 1
This is a scenario when the merger is horizontal without any special conditions influencing the target or the acquirer. Here we are implying that the merger occurs for at least one of the three goals, to implement cost cutting measures, to gain market share, or to reduce competition.

$$K_1 = M(T)$$
$$K_2 = M(T)$$

- $R_1: 0 < R_1 < 1 - M(T)$, Scope of Variance $(0, 0.2)$
- $R_2: M(T) - 1 < R_2 < 0$, Scope of Variance $(-0.2, 0)$

K1 and K2 are assigned $M(T)$ because here we are assuming the target will always seek to gain premium over the market’s view and the acquirer will always try to pay a discount to the market’s view. $R_1$ is the premium target will ask for and $R_2$ is the discount the acquirer will ask for.

Scenario 2
This is a scenario when the acquirer is aggressively bidding for the target. Such situation can occur when the acquirer attempts to fend off future competition, acquire a specific asset of the target for strategic reasons, or values the synergy effect much more optimistically than the target in addition to all the given conditions of scenario 1.

$$K_1 = M(T)$$
$$K_2 < 1$$

- $R_1: 0 < R_1 < 1 - M(T)$, Scope of Variance $(0, 0.2)$
- $R_2: M(T) - 1 < R_2 < 0$, Scope of Variance $(-0.2, 0.4)$

K2 is relaxed assuming the acquirer is willing to pay up to a 40% premium to execute the deal.

Scenario 3
This is a scenario when the target is searching for a buyer. Such situation can occur when the target is at the brink of a bankruptcy but still has quality assets and business or values the synergy effect much more highly than the potential buyers in addition to all the given goals of scenario 1.

$$K_1 > 0$$
$$K_2 = M(T)$$

- $R_1: 0 < R_1 < 1 - M(T)$, Scope of Variance $(-0.4, 0.2)$
- $R_2: M(T) - 1 < R_2 < 0$, Scope of Variance $(-0.2, 0)$

K1 is relaxed assuming the target is willing to sell down to a 40% discount.

Scenario 4
This is a scenario when the acquirer is dishonest about its valuation. Such event can occur when the target is looking to sell and the acquirer is trying to take advantage of the situation in addition to all the given conditions of scenario 1 and scenario 3.
K1 > 0
K2 = M(T)
R1: 0<R1<1-M(T), Scope of Variance (-0.4, 0.2)
R2: M(T)-1<R2<0, Scope of Variance (-0.5, 0)
K1 is relaxed assuming the target is willing to sell down to a 40% discount.

 Scenario 5
This is a scenario when the target is dishonest about its valuation. Such event can occur under the provided environment in scenario 1 and scenario 2.
K1 = M(T)
K2 < 1
R1: 0<R1<1-M(T), Scope of Variance (0, 0.5)
R2: M(T)-1<R2<0, Scope of Variance (-0.2, 0.4)
K2 is relaxed assuming the acquirer is willing to pay up to a 40% premium to execute the deal.

 Scenario 6
This is a scenario when both parties are dishonest, but understand that the deal can generate value for both parties.
K1 = M(T)
K2 = M(T)
R1: 0<R1<1-M(T), Scope of Variance (0, 0.5)
R2: M(T)-1<R2<0, Scope of Variance (-0.5, 0)

Conclusion
We ran our simulation of 12,111 random samples replicating the yearly average number of transactions worldwide. The simulation resulted in the following calculations.

Scenario 1: premium of 0.06%.
Scenario 2: premium of 9.721%
Scenario 3: discount of 9.777%
Scenario 4: discount of 15.401%
Scenario 5: premium of 15.5521%
Scenario 6: premium of 0.213%

Even though this simulation is an extremely generalized version of relative pricing and agreement, we found the outcome helpful searching for the next step of our research. What we found from this test is that the procedure towards agreement can significantly help companies value the deals more accurately. As the results of Scenario 3, 4 and 6 implied, a certain procedure towards agreement prevented significant amount of mispricing even at the event of cheating. We believe the results of Test 1 address one of the most common causes of a failed M&A, overpaying for the target.
3. Fair Division

Here we attempt to describe a generalized language for denoting a Player’s opinion of values of the Target (t), Acquirer (a), and combined value of Both (B) after acquisition and merger.

**Notation**
- T = target player
- A = acquirer player
- t = target cake
- a = acquiring cake
- B = combined cake post transaction
- T(t) = target’s honest value of the target’s value (himself), etc.

**Assumptions**
There are absolute bounds on the deal: There is a minimum value below which T will not sell, and a maximum value above which A will not pursue the deal.

**Development of Negotiation Model**

**Target’s honest opinions of value:**

Define constant \( k \) such that

\[ k \times T(t) = T(a) \]

\[ k \geq 1 \] (Target is never bigger than the acquirer)

In B, Target is envy free if it gets \( \geq 1/(k+1) \times T(B) \)

**Goal:** T wants to maximize \( k \)

Lower bound of the deal range: \( T(\text{min}) = T(t) = T(a)/k \)

**Acquirer’s honest opinions of value:**

Define constant \( s \) such that

\[ s \times A(t) = A(a) \]

In B, Acquirer is envy free if it gets \( \geq s/(s+1) \times A(B) \)

**Goal:** A wants to maximize \( s \).

Upper bound of the deal range: \( A(\text{max}) = A(t) = A(a)/s \)

Redefine \( S \) in terms of the Target:

\[ 1 - (s/(s+1)) = m \], where \( m \) is the max ratio Acquirer is willing to give target

\[ m = 1/(s+1) \]

Now, Acquirer is also envy free when Target gets \( \leq m \times T(B) \)

**Goal:** A wants to minimize \( m \).

Deal Range = \( A(\text{max}) \rightarrow T(\text{min}) \)
An Honest Assessment of perceived value

**An Arbiter to Evaluate Fairness:**

Upon the beginning of bargaining, each Player is only privy to his personal views. However, the arbiter will know both Players’ views, and can accurately assess fairness of a deal. The arbiter will also have an additional constant, \( d = \frac{A(b)}{T(b)} \). Since Players’ views of the new combined \( B \) may not be equal, and since the value of \( B \) is not guaranteed to be additive, this normalization constant is important to have in assessing the fairness of the deal.

Future Research: We plan to continue investigating how initial Players’ views of \( T, A, \) and \( B \) and the ratio \( d \) give advantage to one party over another.

**Bargaining Positions:**

Each Player has its own unique opinion about the value of \( t, a, \) and \( b \). The process to reach agreement on the value with which the deal will happen, in essence the ‘Target’ stake in the new entity \( b \), will take place through some sort of negotiation process. In this process, each Player wants to maximize its return from the deal. We can define the starting bids of these Players as \( k' \) and \( m' \).

In the above scenario tests, we defined \( k' \) and \( m' \) in terms of random distributions within given around an additional simulation variable, real market value (RMV).

This allowed us to investigate the implications of deals given different beliefs about the worth of the Target, and see who was advantageously favored given a simple median agreement, dividing the new combined entity \( B \) as would be done in Austin’s extension.

Future Research: We would like to continue to investigate the implications of these initial valuations by \( T \) and \( A \) as compared to the bounds of the deal, since \( m' \) and \( k' \) are not guaranteed to follow within \( m \) and \( k \). This would provide insight into the feelings of the Player who negotiates outside the bounds, and would change the other Player’s negotiation tactics, or possibly end the negotiation without a deal.

The simulations created all starting positions to ensure a deal, and presented a simplified bargaining model to represent each party acquiescing toward a mutually agreeable solution in fixed step sizes, in essence finding the median between the two views.
However, once parties began exaggerating their views of the value of B, cake cutting algorithms are no longer applicable, since they cannot fairly divide resources when their true value is unknown. (The RMV in the simulation represents the value of the deal known afterwards, and is information not necessarily present in the deal).

**Adding Additional Constraints to Emulate the Real World:**
As of current, we have developed a model to be able to describe this abstract problem of quantifying the viewpoints and constraints of a merger or acquisition. To do this, we relied upon literature surrounding Cake Cutting Theory from the class Special Topics in Computer Science Theory at Washington University in St. Louis, and through investigation of the principles of automated bargaining agents in published research.

To make this model more applicable to real world situations, we added the following constraints to the model described above:

- **Time Cost:** a Player’s sensitivity to time, limiting the duration he can stay in the negotiation process
- **Deadline:** a Player’s maximum amount of turns before he must make a deal
- **Residual Value:** cost or value left with a Player when the deal falls apart
- **Arbitrary Closing (AC):** of the deal after an unknown number of negotiations. This prohibits a Player from taking advantage of the opposite Player’s need to make a deal within a certain timeline or at all, and stalling until the other Player is forced to accept its offer.
- **Player (dis)advantages of the deal:** This would reflect a higher or lower value than the additive sum of A + T. This would take into account a Target’s need to sell, (pending bankruptcy, etc.) for which it would accept less than equal value to its own view of itself, or an Acquirer’s strategic advantage gained from T, which might be greater than the summation o A + T. Both examples change the bargaining strategy for the part involved.

We also would like to introduce preferences into the step size of negotiations taken by each Player to reflect the initial bid of the acquirer and subsequent bid of each Player. This would reflect where the other Player’s bids lie within that Player’s view of the world.

Additionally, reflecting multiple Player’s views on each other’s value when B is unequal between their views provides an interesting inequality, which further makes traditional cake cutting models difficult to implement, especially in regard to an equal solution. For example, how does one find a fair solution when none exist because users exaggerate their views? This is especially difficult in situations in which there is no market value. The arbiter can declare a mutually agreeable solution since it knows both players true preferences, but this is unrealistic for this type of problem.

We plan to continue this research into next semester to develop an automated, flexible/learning, bargaining model to address this problem and find the fairest solution in an environment with
cheating and lack of information. We believe a traditional cake cutting approach is difficult to implement to produce an envy free and fair solution, but as our initial Test implied there can exist an adjusted procedure that can provide a cake cutting approach to M&A transactions.

The modeling language developed in our research provides a concrete manner in which to investigate the generally abstract topic of mergers and acquisitions, and provides a bounded set of constraints within which these deals will happen. The actual method of finding the fairest deal between two or more parties requires further investigation, and will be continued through next semester. Our final goal next semester is to further develop our model and gather valid data to test and compare our methodology.
Reference

1. Course 7: Mergers & Acquisitions (Part 1) Matt H. Evans, CPA, CMA, CFM, Published March 2000